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DIFFERENTIATION BY DEFORMATION

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Introduction.—The deformation of an igneous mass during crystallization, with consequent separation of liquid from crystals, has frequently been suggested as a cause of variation of igneous rocks but the suggestion has apparently never received as detailed consideration as it warrants. That variation would result is obvious, but the full extent of its importance will not be realized unless one has an adequate picture of the far-reaching consequences that may follow from the separation of crystals and mother liquor. In another paper the writer has demonstrated the consequences of such separation in artificial melts and applied the results to natural magmas.[1] Emphasis was there placed upon gravity as a means of separation of crystals from liquid but the chemical consequences are identical whatever the means of separation. In certain respects, however, the consequences of separation by deformation may be distinctive and to these attention is directed in the present paper.[1]

Discontinuous differentiation.—Discontinuous variations, in particular, are not normally to be expected as a result of gravitative accumulation of crystals but discontinuity would appear to be a very likely consequence of deformation. This is especially true at a late stage of crystallization when deformation may cause a crushing of the crystal mesh with consequent closer packing of the crystals and onward movement of the interstitial liquid. At first thought it might seem that the liquid could not move without carrying the crystals with it but it may be pointed out that the break down of a crystal mesh is likely to be progressive and any area breaking down will in such circumstances be bordered by an area not yet affected. The residual liquid is, therefore, free to move into the interstices of the unaffected portion of the mesh and to drive ahead of it the liquid already there which may become a separate intrusive body or a distinctive portion of the same intrusive and of late consolidation. Elsewhere the writer has offered the suggestion that the common association of gabbro and granophyre is frequently capable of interpretation in this

manner and perhaps many other associations particularly where the differentiation is markedly discontinuous both chemically and spatially.

Monomineralic types as members of composite intrusives.—Differentiation as a result of settling of crystals and as a result of deformation are by no means mutually exclusive; indeed, it seems probable that in one case, in particular, deformation may step in to bring to completion a task begun by gravitative accumulation, namely, in the case of the production of monomineralic types of excessive purity. For anorthosites, by way of example, it is probable that, besides the sorting of plagioclase crystals by gravitative action, a squeezing out of the interstitial liquid may have contributed to their extreme purity in many cases. If the squeezing continued after a high degree of compaction of crystals had been obtained a clastic structure might be developed and the common protoclastic structure of anorthosite may perhaps be interpreted as evidence that a squeezing out of residual liquid has played an essential part in its production.

Monomineralic types as simple "intrusives."—Peridotite as a sill-like "intrusive" would appear to be capable of production by a somewhat similar combination of crystal settling and deformation. If the intrusive material was originally simply basaltic, if in early stages of crystallization the settling of olivine crystals to the floor was pronounced and if at this time deformation occurred such that in certain places the roof was down-warped the result might be that the whole width of the sill in these places would be occupied by peridotitic material. The down-warping of the roof at this early stage of crystallization might not only remove the supernatant liquid from above the accumulating olivine crystals but might be of sufficient vigor that the interstitial liquid of the peridotite should be largely expressed and a mass of typical dunite result. The mass of peridotite or dunite so formed would have all the ear-marks of an ordinary intrusive sheet and there would be no internal, textural evidence that it had not crystallized from a molten mass of sensibly its own composition though no peridotitic or dunitic liquid ever existed in that region. The formation of this peridotitic differentiate connotes the possibility, indeed the probability, of a complementary granitic differentiate which may be represented as one member of a composite sill in those parts of the sheet that have been widened during the deforming action. The association of peridotitic intrusives with composite intrusives of this type is rather common and merits examination on the part of field-workers with the above suggestion as to origin in mind. In the asbestos region of Quebec the peridotite (now serpentine) masses are described by those best acquainted with them in the field as synclinal sheets.[2] This designation together with the association of diabases, diabase breccia, and granitic intrusives, suggests that the region may furnish a concrete example of the origin of a sheet-like peridotite "intrusive" after the manner described above.

It seems possible that even a dike-like mass of basaltic magma if deformed at an early stage of crystallization, might give rise, in the portions that are constricted by deformation to a peridotitic mass, though probably not as readily as in the case of a sheet. The mutual approach of the walls necessitates a movement of liquid out of the intervening space but close to the walls there would be a certain amount of drag of the crystals resulting in some movement of liquid relative to crystals. The two marginal regions of concentration of crystals might be brought together and even crystal packing and the expressing of liquid might occur if the constriction were carried far enough. Thus might result a dike which, intersected at this particular point, would be simply a peridotite. It would, therefore, seem necessary to include the above action among those to be considered as capable of producing dike "intrusions" of peridotite.

Complementary dikes.—Those minor but rather constant associates of intrusive masses, the complementary dikes, aplitic and lamprophyric, may perhaps be produced by action of the same general nature as that we are now considering. In very late stages of crystallization residual liquid may be drained into clefts in the mass and there suffer differentiation as a result of local pinching and swelling during crystallization. This is particularly likely to have important effects in such small bodies. Complementary units would be formed that might readily have a composition not matched in large intrusive masses.

Primary banding.—A further effect of deformation during crystallization appears to be that which finds its expression in primary banding. Shearing of a crystalline mesh, particularly when it is still weakly knit together and permits passage of liquid through it with great freedom, may develop a feature of the same general nature as a shear zone developed in solid rocks. Lenticular openings may be formed that instantly fill with liquid from the interstices of the mesh and repetition of the action may give rise to a banding, properly oriented with respect to the shearing stress, and showing approximately that contrast between bands that obtained between liquid and crystals at the time of the action. The crystals that became detached during the shearing would naturally become aligned in the liquid filling the lenses so that fluxion structure would be a natural accompaniment. Moreover, in the larger lenses developed a gravitative sorting of these detached crystals would take place under particularly favorable conditions and bands showing the extreme contrast of monomineralic types might thus be formed. The banded gabbros exhibit features that appear to match the results of the postulated action. Other rock types should be subject to similar action, but are probably not as likely to give such obvious contrasts in the bands as are the gabbros.

Alkaline rocks.—In rare instances rocks are found that have empty interstices and to these the term miarolitic is applied. It is usually assumed in explanation that the rock had crystallized in such a way as to

leave only gas as interstitial material. This is, however, not the only possibility. It is not difficult to conceive of a distribution of forces in the heterogeneous surroundings of an igneous mass such that locally the liquid might be sucked out of a crystalline mesh, which would acquire a miarolitic texture, temporarily, at least. The consequences of this action would normally differ in no essential particular from those produced by simple squeezing out of the liquid. It is probable, however, that this sucking action could be operative at a very late stage of crystallization when squeezing out of liquid may be impossible or at least very unlikely. The kinds of liquid that form some of the alkaline rocks may perhaps be removed in this manner from quartz-mica rocks at a very late stage of their crystallization and possibly only such action can effect the separation of these liquids. In certain regions of the earth's crust where tangential extension is the dominant expression of the forces acting (Atlantic structures) the development of alkaline rocks might be a prominent feature though the conditions requisite to their formation would undoubtedly occur locally elsewhere.

[1] The Later Stages of the Evolution of the Igneous Rocks, *J. Geol.*, **23**, 1915, Suppl., pp. 1–91.

[2] Howie, Robt., Summary *Rept. Geol. Survey Can.*, 1916, Coleraine Map Sheet insert, p. 228; also Knox, G. K., *Ibid.*, p. 229, et. seq. Reference is made to granitic "batholiths" in the anticlines, p. 232.

THE EVIDENCE FOR THE LINEAR ORDER OF THE GENES

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Communicated February 25, 1920

Despite Castle's dictum that we "have failed in two different attempts to establish the linear theory in the case of the three genes yellow, white and bifid," we are bold enough to maintain that the data furnished, and still furnish, the proof called for. We wish to call attention to the fact that in his last paper Castle ignores our proof of the linear order that is furnished by building up the whole chromosome (or even large sections of it) by "distances" so short that no double cross-over classes appear.

Castle asserts that we have rejected "nearly 99 per cent" of our data in the construction of the yellow, white, bifid section of the map. As a matter of fact no data have been omitted. In this case, as always, the order of the loci was determined by experiments that involved all of these loci at once. The order having been established the next step was to determine the relative distance between the loci by the use of all the available data. We have emphasized in our reply to Castle that there are several sources of variability in linkage values such as age, temperature, genetic factors. The variability due to these causes far outweighs that due to random sampling. It is, therefore, inadmissible to compare data from different

experiments, however extensive, in establishing the order of the loci, or in testing the validity of the hypothesis of linear arrangement. For such purposes, as we have already pointed out, it is essential to use data in which all the loci are followed at once.

The purpose of the maps is twofold: first, to give the sequence of the loci, and secondly, to indicate, by the relative spacing of the loci, the cross-over values most likely to coincide with the results of future experiments. For the latter purpose it is evident that mean values are needed. These can best be obtained by using all the data. And this, as stated, has been the method used for determining the distances on the maps published. Castle has used all the data for criticizing the sequence of the loci on our maps, and has accused us of using only the three-point data for determining the map-distances. Obviously this is an inversion of the correct relationship.

Even when all the available data are used in constructing the map, these data should themselves be subjected to all the controls that it is possible to apply to them, as everyone familiar with the treatment of rough data will understand. For example: It is well known that under certain conditions, such as crowding, some mutant classes run behind their expected ratios. Under favorable conditions of culture, normal ratios are obtained. Clearly, data of the first kind unless corrected are unsuited for determining distances on the chromosome map; and the larger the amount of such data, the larger would be the discrepancies in a map based upon such data used with normal data. We have shown how the aberrant ratios due to the inviability of given classes can be balanced by the use of converse crosses. Further corrections and weighings are also desirable in order that no one class of data shall unduly prejudice the result. The methods for making such corrections have been briefly outlined in Carnegie Publication No. 237, 1916, and given more fully in Carnegie Publication No. 278, 1919.

Besides environmental disturbances of the kind just described there are factors that are known to modify crossing-over. Obviously such data should be eliminated from material from which a normal chromosome map is to be made. That Castle himself admits the validity of elimination of such data is shown by the fact that he deliberately rejected the data involving lethal 2 which we had shown to cause aberrant linkage relations. If he had used these data, all of his long wires would have been bent. Castle has set up the claim that one of the advantages of his model is that such a bending of a wire in the case of white forked revealed the fact that the experimental value of this interval was too great. The same fact would have been "revealed" on any system by a comparison of the white forked value with the values for white rudimentary and white bar, both of which were based on larger numbers than white forked itself.

Castle's discussion of interference in connection with yellow white bifid (p. 503) shows his failure to understand interference, and hence his criti-

cism misses the mark. It need only be stated that Castle was wrong in supposing that the cross-overs between yellow and white should be subtracted from the total before the white bifid cross-over value is calculated.

If, as Castle states, he cannot conceive of a mechanism "which would tie two genes together in such a way that they will subsequently separate from each other oftener than they will remain together, yet this is what the idea of cross-overs in excess of 50 per cent amounts to," it would seem to follow that he has not really understood the mechanism that we have described, and which he has attacked; for, whether such a mechanism really exists or not, it is nevertheless a conceivable mechanical device that could do just this thing.

We have left to Dr. Metz the opportunity to answer Castle's criticism relating to D. virilis.

To sum up: we believe that we have met all the pertinent criticisms that Castle has brought forward of our methods and conclusions, and that he has failed to meet our criticism of his three dimensional model.

THE ARRANGEMENT OF GENES IN DROSOPHILA VIRILIS

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Communicated by T. H. Morgan, March 6, 1920

In connection with his general attack on the theory of the linear arrangement of genes Castle[1] has questioned my use of this theory in explaining the genetical results obtained in Drosophila virilis. I did not consider it necessary to reply to this criticism because Castle's general position was shown by Sturtevant, Bridges and Morgan[2] to be untenable as regards Drosophila melanogaster, with which D. virilis agrees in its mode of inheritance. Subsequently, however, Castle has reaffirmed his belief in the superiority of his hypothesis over that of linear arrangement[3] and has apparently misconstrued the silence on my part with respect to Drosophila virilis. This would seem to call for a brief reply.

From my paper on eight sex-linked characters in D. virilis[4] Castle concluded that the genes dealt with could not be arranged in a linear series, and by applying his three dimensional hypothesis to the case he made certain predictions (*b*, p. 36) which he now wishes used as a test of his hypothesis. His general arguments in this case are the same as those he used previously in reference to Drosophila melanogaster, and since they are being treated by those toward whom they were first directed, I will avoid repetition by confining myself to his predictions. These apply to four undetermined cross-over values in D. virilis, namely, magenta-hairy, glazed-rugose, frayed-forked and frayed-glazed.

It should be pointed out first that in making these predictions Castle apparently overlooked the statement in my paper (pp. 113 and 125) that the frayed stock had been lost shortly after it was obtained. The absence

of this stock makes it impossible to test the latter two of his predictions. The other two predictions are also such that they cannot be tested at present; the first because of the loss of hairy stock through sterility, and the second because of the sterility of rugose-glazed hybrids.[5] Had it not been for these practical obstacles the values would not have been left undetermined in my original paper.

The matter need not be dropped here, however. It should be noted that, in any event, tests of the predictions would have been futile, for the two hypotheses may be compared just as well by means of known values as by testing unknown ones. In addition, the very nature of the predictions themselves prevents their being used for critical tests, because they are so constructed that, on either view, they must be fulfilled if my data give sufficiently accurate ratios.[6] There is no alternative. This being the case it becomes a question of ascertaining the accuracy of my data[7] or the validity of the assumptions underlying Castle's predictions.

An illustration may be used to amplify this point. Castle's last two predictions apply to the gene for frayed. My data on frayed (pp. 112 and 126) as tabulated by Castle would locate the gene 1.3 units from yellow and 18.6 units from vesiculated. Castle's own calculation of the value yellow-vesticulated is 17.4. This would put the three genes in almost exactly a straight line (1.3 + 17.4 = 18.7, as compared with 18.6). These are the only data available for the direct determination of the location of frayed, but so far as they go they conform to the linear hypothesis. Castle's predictions relate to the cross-over values that should be given by frayed and forked, and by frayed and glazed. The former should be "between 39 and 41" and the latter "between 43 and 46." The predicted values are, of course, to be calculated entirely on the basis of single cross-overs, as all values are determined on his system.

These predictions are evidently based on the cross-over values given by yellow and forked, and yellow and glazed, respectively. By leaving out of account the double cross-overs, Castle calculated the latter values from my data as 40 and 44.5. Since frayed is approximately one unit from yellow it should give within approximately one unit of the same cross-over values as yellow. This principle would apply on either hypothesis. It is evident, then, that by calculating the value frayed-forked in the same manner that yellow-forked was calculated—namely, by using only single cross-overs—the result would have to be within approximately one unit of 40, if crossing over is a consistent process. The same principle applies to the frayed-glazed relation. Both predictions would have to be fulfilled (within the limits of experimental error), under these conditions. But this does not in any way substantiate the three dimensional hypothesis. Predictions in such cases as these can be made and fulfilled on either hypothesis, providing the calculations are made in accordance with the hypothesis (the one including, the other excluding, double cross-overs).

It is not necessary to carry the analysis farther, since neither theory would be benefitted by the fulfillment or non-fulfillment of these predictions. The question at issue is not met in this manner. It is involved in features lying outside the province of the present paper and is being treated by other authors, as mentioned above.

Castle, W. E., (*a*) "Is the arrangement of the genes in the chromosome linear?" these PROCEEDINGS, **5** 1919 (25–32); (*b*) "The linkage system of eight sex-linked characters in Drosophila virilis (data of Metz)," *Ibid.*, **5** (32–36).

[2] Sturtevant, Bridges and Morgan, "The spatial relations of genes," *Ibid.*, **5**, 1919 (168–173).

[3] Castle, W. E., "Are genes linear or non-linear in arrangement?" *Ibid.*, **5**, 1919 (500–506).

[4] Metz, C. W., "The linkage of eight sex-linked characters in Drosophila virilis," *Genetics*, **3**, 1918 (107–134).

[5] See Metz and Bridges, these PROCEEDINGS, **3**, 1917 (673–678). Before the hairy stock was lost some data on the value hairy-magenta were obtained by Weinstein, as noted by Muller (*Amer. Nat.*, **54**, p. 118), but the numbers were hardly large enough to be conclusive.

[6] Assuming, of course, that crossing-over is a consistent process, and that the experiments are conducted under similar conditions.

[7] My data, in the cases under consideration, include only relatively small numbers and give ratios of only approximate accuracy, as indicated in my paper. This fact alone would vitiate predictions of short distances (such as the first two) even if they were otherwise valid.

THE FUNCTIONS OF INTENSITY AND PHASE IN THE BINAURAL LOCATION OF PURE TONES

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Communicated by R. A. Millikan, February 5, 1920

The permanence of the value of knowledge concerning the physical aspects of audition gives to contributions in this field a greater importance that is at first apparent. There has been much confusion as to the factors involved in the binaural location of sounds. An analysis of the problem of ascertaining the important factors and their relative values show that the solution will be secured most speedily by the separate control of these factors and by using first pure and then complex tones. This report includes experiments covering the frequency range of 100 to 1200 d.v., the results giving the effects of intensity differences and phase differences at the ears, each factor treated first alone and then in combination.

Intensity.—Rayleigh (*Phil. Mag.*, **13**, 1907 (214)) was the first to point out that for frequencies of 128 and 256 d.v., the difference in intensity at the ears could not account for the ability to locate the source of sound. Although there have been many experimental contributions in the localization of sound, the only observers to make quantitative measurements of the effect of intensity were Hovda and the writer (*Psych. Rev.*, **25**, 1918,

3 (242)). They found that, for a frequency of 256 d.v., if the difference of phase at the ears was kept zero but the ratio of intensities varied, there appeared a moving phantom source having a position in front, for an intensity ratio of unity, and rotating horizontally about the observer, its angular displacement from the median plane being proportional to the logarithm of the ratio of the intensities at the two ears, the constant of proportionality varying for the individual and for the frequency. This logarithmic law has now been further tested for frequencies of 512 and 1024 d.v., and has been found to be correct for displacements from the median plane, less than 90°, excepting that for the 1024 d.v. some individuals do not get a moving phantom source of the fused sound. It would thus appear that intensity might have an important influence in localization. But such a conclusion must involve a quantitative test. Assume a source of sound at a distance of five meters from the observer and carried in a circle about him. It is possible, by means of theory (Stewart, *Physic. Rev.*, **33**, 1911 (467)), to compute the ratio of intensities at the ears produced by such a source at different angular displacements from the median plane. But one can, by means of the experiments already mentioned, ascertain what apparent angular displacements would be produced by the computed intensity ratios. Do these computed displacements agree with the actual? If they do, obviously intensity is an important factor in the actual case. But they do not agree quantitatively. For example, if the displacement of the source was 60°, the computed displacements for one observer were as follows: for 256 d.v., 6.1°, for 512 d.v., 13.8°, and for 1024 d.v., 11.2°. For a second observer, and the last two frequencies, the computed angles were 9.1° and 9.1°. A careful examination of all angular displacements up to 90° showed a similar lack of correspondence. Obviously the intensity effect cannot account for the ability to locate sounds. It does not appear to be even an important factor.

Experiments showed that for some people (two out of four observers) the intensity effect disappeared entirely in a certain frequency region. With each of four observers the intensity effect in a relatively large frequency region involved not only a moving phantom source of sound, but also an additional one stationary and in front, or in a position corresponding to the difference of phase.

The logarithmic law of Stewart and Hovda appears not to be merely an extension of Weber's Law, as at first announced.

Phase.—That phase difference affects the localization of a sound source, has been known for a number of years. (Review of literature; Stewart *Physic. Rev.*, **9**, June, 1917 (502).) Hartley (*Physic. Rev.*, **13**, 1919 (373)), has produced evidence in favor of the conclusion that the phase difference of a pure tone at the ears is the controlling factor. The present report presents a considerable addition to this evidence and that of a quantitative character. In the experiments, frequencies from approximately

100 to 1200 d.v. were used. Purity of tone was approximated in a special apparatus, but the results were checked by tones from tuning forks and hence the conclusions are reliable. It was found that, if the phase differences at the two ears were varied, intensities remaining equal and constant, the angular displacement of the phantom sound from the median plane was proportional to the phase difference at the ears. Inasmuch as the theoretical phase differences at the ears were known (Stewart, *Physic. Rev.*, **4**, 1914 (252) and Hartley, *Ibid.*, **13**, 1919 (373)), it became possible to ascertain whether or not the relationship between phase difference and angular displacement was quantitatively the same as in the experiments just mentioned. When the comparison was made, it was found that there was a reasonably close quantitative agreement, i.e., the difference over practically all of the range was, on the average, less than 10%. In fact, the correspondence between results in the theoretical case and in the experimental one was so striking as to leave no doubt as to the fact that *phase* must be the *most important* factor in localization in the region 100 to 1200 d.v.

As can be readily understood, the higher the frequency the less that displacement from the median plane for which the phase difference is 180°. For 1024 d.v., this displacement is 41°. But this is not the maximum angle for which a source can be located by phase. This was shown by the existence of multiple phantoms or images. Thus the conclusion already stated seems worthy of full confidence as applicable for a displacement of less than 90° in front. This does not close the case with pure tones for there is yet to be ascertained what additional factors are of moment in locating a source not in the horizontal plane and not in front of the observer.

That it is impossible to explain the difference of phase effect in terms of intensities, either by bone conduction or otherwise, is proved by the fact that the difference of phase effect was clearly present at 1024 d.v., with two observers who could not at this frequency get the intensity effect at all, i.e., the moving phantom source occasioned by altering intensity ratios was not present. The necessary conclusion is that the ears do perceive phase differences.

The difference of phase effect ceased with increasing frequency at a frequency of 1200 to 1500 d.v. This was tried with four observers. When the effect ceased the phantom sound remained in the median plane with changing phase.

Intensity and phase combined.—Experiments in which phase and intensity differences were simultaneously applied showed that the actual relative importance of either of the two does not change in the combination. Thus the effect of an intensity difference at the ears merely made a relatively slight change in the angular displacement of the phantom source. Consequently our conclusions formed by phase difference altera-

tion, only, are approximately correct for the actual case of location wherein both phase and intensity are altered.

The work thus far clearly shows that with frequencies 100 to 1200 d.v phase is the *chief* factor in localization with pure tones. For the lower frequencies the intensity effect is practically nil, and at all frequencies within the range considered the effect must be small. These conclusions are quite contrary to the view commonly held and appearing in psychological literature and texts. But they are worthy of confidence because based on *quantitative* results, and not on personal judgment. There remains a great amount of experimental work covering an extension of the frequencies and the employment of complex tones. Strictly speaking the localization herein mentioned applies only to 90° either side of the median plane and in front. The removal of this limitation will probably prove relatively simple after the experiments are completed. The above report while covering a limited range, nevertheless does make a definite advance in our understanding of the relative importance of the factors, phase and intensity, in binaural localization, and furnishes a basis upon which the future experiments may depend.

THE MEASUREMENT OF SMALL TIME INTERVALS AND SOME APPLICATIONS, PRINCIPALLY BALLISTIC[1]

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Communicated by A. G. Webster, February 25, 1920

Precision in time calibration is of importance in many classes of research. Several types of apparatus have been developed for this purpose. Some of these, such as the Helmholtz pendulum, permit extremely accurate reproduction of a stated interval but less exact absolute determination. A method proposed and extensively used by A. G. Webster has given satisfactory, absolute determinations of intervals as small as a millionth of a second.[2] The recording apparatus is electrical and the calibration is accomplished by a careful measurement of the height of fall of a projectile.

The Le Boulengé and the Aberdeen chronographs are in general use, the former having a falling projectile and the latter a rotating drum. With any apparatus involving a rotating drum, it is necessary to maintain the speed of the motor driving it within certain limits and to secure a dependable calibration, usually placed on the record as it is being made. Various instruments have been devised to accomplish this purpose.[3] A tuning fork producing a photographic or mechanically inscribed record of a sinuous form is perhaps the commonest example.

Means for applying discontinuously a record of a fork or other vibrator

have been used. For example, a very successful method was recently developed at the Bureau of Standards by Duncan and Curtis in which straight lines were produced across the photographic film by the interruption of light passing through slits attached to the tuning fork.

In the ballistic investigations which have been conducted at Clark University during the past two years, a great many of the records obtained have required such a time scale; for example the pressure-time curves for a gun, the curves for the vibration of the gun barrel, and later the velocity measurements which are described in this paper. The Bureau method was first adopted. We have been able, however, to develop a new apparatus which permits the placing of equally sharp lines across the film at very high frequencies. This is of considerable importance in the small arms ballistic work, inasmuch as it is necessary to have the film moving at high speed.

The apparatus for the time records is shown in figures 1 and 2. The tuning fork has a small needle attached to one prong (a weight being attached to the other) against which there bears a light stiff filament of steel ribbon held in place by a massive frame attached to the base mounting

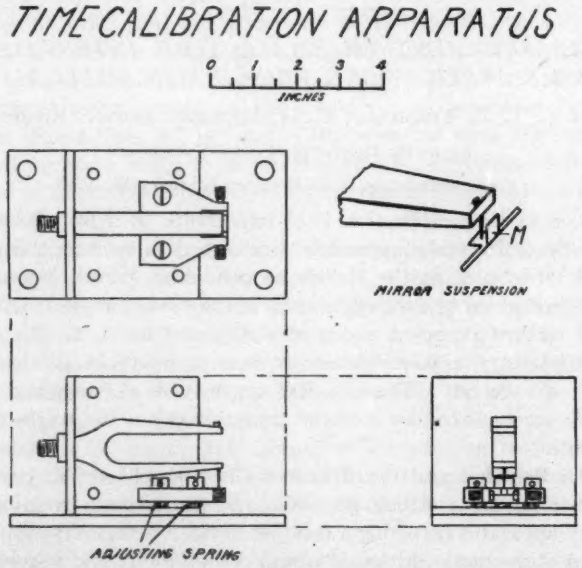


FIG. 1

of the tuning fork. (The filament is practically a duplicate of the one used in Professor Webster's Phonometer to record the motion of the vibrating disk.)[4] Upon this filament is mounted a small mirror. Light is reflected from it and focused on a stationary slit. The fork is struck with an elec-

trical hammer and allowed to vibrate freely, and in so doing causes the image of the slit in front of the arc to move up and down across the stationary slit. The stationary slit, then, is an intermittent source of light, the image of which is focused at a convenient place on the moving film. The optical system should be so designed that the beam only slightly more than covers the mirror. A new fork has been constructed giving about two thousand lines per second. There is no reason why this should not be as high as ten to

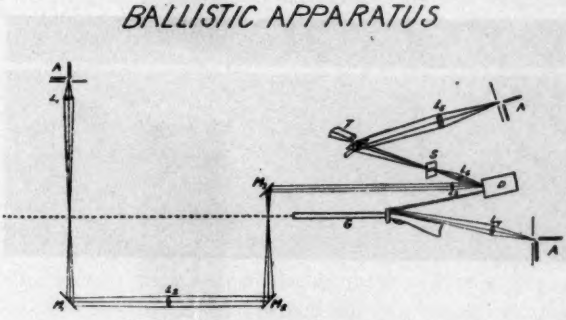


FIG. 2

twelve thousand lines per second. The velocity at the neutral position (where the slit is placed) is a maximum and includes the frequency as a factor. As it is increased the amplitude may decrease and still permit the same exposure of light through the slit as at the lower frequencies. Mechanically, the essential condition to be fulfilled in the design is that the natural frequency of the filament upon which the mirror is mounted should be high in comparison with that of the fork.

Most of the methods which are at present used in ballistics for the measurement of velocity involve the passage of the projectile through screens of one sort or another, breaking or making electrical circuits. The method which we have devised does not require that any material be touched by the projectile in its flight, but simply the passage twice through a beam of light. It is found that it is only necessary to observe the flight of the projectile over a space of from 3 to 5 feet to get a precision which is certainly greater than that needed for ordinary purposes.

The beam of light obtained from a slit is controlled by the optical system (fig. 2) in such a way that it is focused twice, each time in the path of the projectile, being reflected by the mirrors M_1, M_2 and M_3 and finally brought to the photographic surface on the rotating drum. The image is a short sharp line. It is desirable to have for both this and the calibrating system a short focus lens. As the film rotates, the image of the slit will

trace a narrow ribbon on the film surface. As the projectile passes through the ribbon in the two places successively an interruption of light occurs and a dark section appears in the ribbon at each of the corresponding instants. This is in general a longitudinally distorted image of the projectile and if the speed of the film were adjusted to a certain value it would be a miniature reproduction of its cross-section. Some records which have been obtained are shown in figure 3. The measurements have been

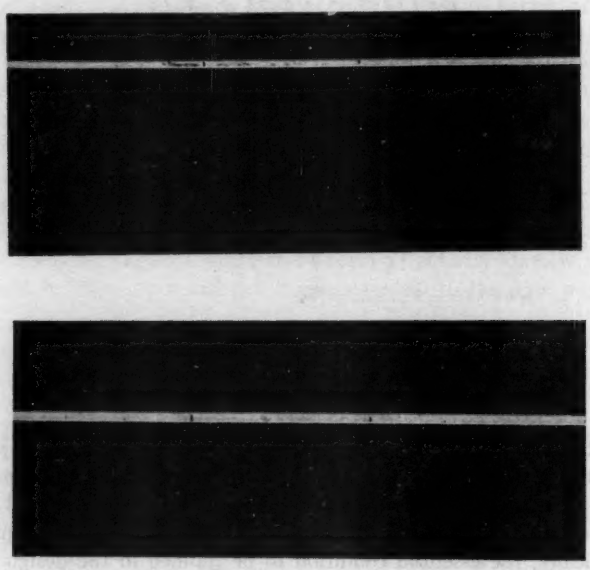


FIG. 3

made from the base line of the projectile in each case and the sharpness of this permits a determination of the length between these lines with a precision which is fully equal to that of the time calibration. The only observations, then, that are required are those of length. Each of these can be carried out to one part in ten thousand if necessary. The fact that a permanent record is provided and that no electrical apparatus is used also adds to the dependability of the results. A compact portable form has been designed for field use.

A general apparatus for ballistic investigation is shown in figure 2. This includes a pressure gauge[5] as well as the velocity equipment. With one discharge of the gun a fairly complete ballistic record is obtained on the film mounted on the rotating drum *D*. The velocity at the muzzle, the pressure-time curve in the barrel, including the pressure when the projectile leaves, that at maximum value, the time calibration, and, by

analysis of these curves, the velocity-time and pressure-space curves, as well as data regarding the resistance of the barrel, are all available.

An extensive application of this procedure is to be made in a determination of the resistance of air at various densities, to the flight through it of bodies moving with relatively high velocities. Heretofore, measurements have been made out-of-doors, shooting over a comparatively long range in order to get the desired accuracy. By making use of this velocity apparatus we are able to make two determinations in a length which is to be obtained in a building of laboratory size. The resistance is measured by the work which is done in the passage of the projectile from one point to another through the medium. This will require the two determinations of the velocity for the change in kinetic energy. It is necessary that the velocity be found with a precision which is sufficient to give the difference in the squares of the velocities with an acceptably small error. For a Government thirty caliber ($^1/_{50}$ lb.) projectile the resistance of the air at normal pressure is such that the velocity will decrease an amount of the order of 225 feet per second in travelling 300 feet from the muzzle. At densities greater than normal the decrease will, of course, be greater and at pressures less than normal (which will be obtained as described below) the decrease will be less than this.

$$\frac{\int_{s_1}^{s_2} R\,ds}{s_1 - s_2} = \bar{R} = \frac{m}{2s}(v^2 - v_o^2)$$

is the mean value of the retardation over the distance of observation (100–200 feet).

An approximate idea of the effect upon the final value of the resistance of an error in the measurement of the velocity of one part in ten thousand is obtained by an examination of the above expression.

$$\frac{\delta R}{R} = \frac{2v\delta v}{v^2 - v_o^2} + \frac{2v_o\delta v_o}{v^2 - v_o^2} + \ldots = \frac{4 \times 2500 \times {}^1/_4}{650000} + \ldots \text{approximately, or about } 0.4\%.$$

Here $\Delta v = 125$; $v_o = 2650$; $v = 2525$; $\delta v = \delta v_o = {}^1/_4$. At higher pressures the error will be still less and at pressures down to one-half normal the substitution in this expression shows that the error for a single observation will still be below 1%.

Inasmuch as the ratio of the resistance of the medium to the weight of the projectile decreases as the diameter increases and in direct proportion, the precision which would be expected with large projectiles would be correspondingly less. Nevertheless, by increasing the distance of observation the same order of accuracy would be possible with projectiles up to perhaps 3 inches in diameter. If the firing is done in a tube of from 1 to 3 hundred feet in length, having a diameter sufficient to make observation possible without great inconvenience, the air pressure in this tube being controlled by a pump, the observations of the

velocities ranging perhaps from 1200 feet per second up to 3500 or 4000 feet per second can be made at each end of the tube. As long as the velocity of the projectile is greater than that of sound, no interference with the measurements will result from a disturbance reflected from the walls of the tube.

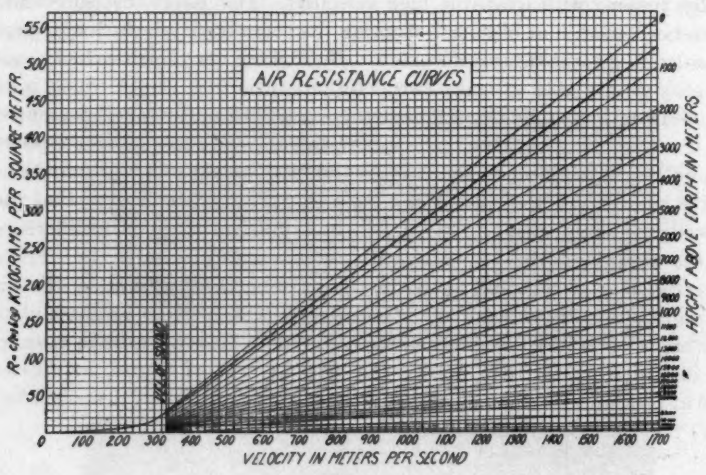
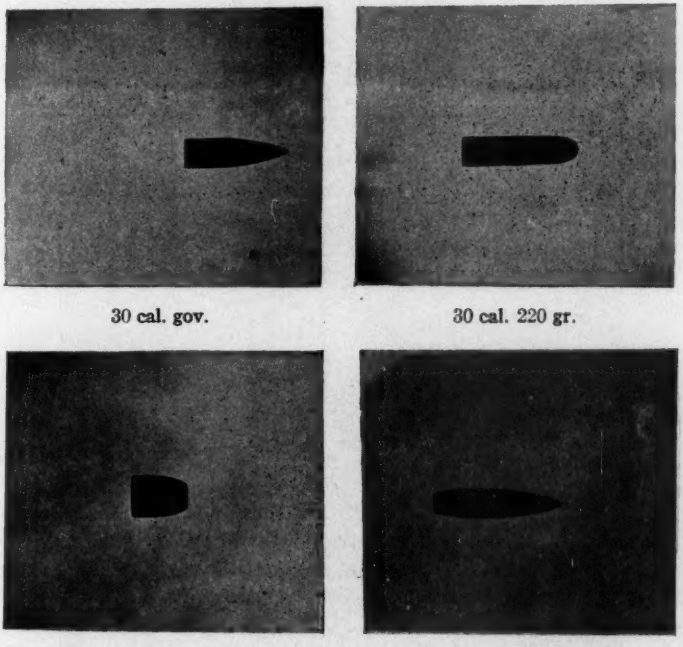


FIG. 4

Figure 4 is a graphical representation of the resistance as a function of the velocity as determined from the measurements made by at least five European observers, their data being averaged by the Italian ballistician, General Siacci. The retardation appears to be a linear function beyond the velocity of sound. Multiplying the datum line of these curves by different factors depending upon the density of the air gives the other curves for various heights. These were used in trajectory computations on long range guns of the type brought out by the Germans to shell Paris.[6] The experiments outlined will be adequate to make a precise determination of the values for the resistance for small projectiles over a considerable part of the usable portions of this graph for the modern rifles of high velocity. The fact that this problem is not the exact equivalent hydrodynamically of the one for very large projectiles does not materially lessen the value or interest of the results of such an investigation with small projectiles.

An important problem is the accurate determination of the effect of varying the form of the projectile. The question of ultimate or optimum form for a given use should be experimentally determinate. The records show not only the velocity but the manner in which the projectile is travelling. Also, it is likely that the precision and the complete control of the conditions of experiment possible would lead to satisfactory results

as to the variation of the so-called form factor with the velocity. If the results indicate an entirely new law of resistance for each form of projectile, as has been suggested, this fact should be evident. In this case there will not be a linear relation between the datum line for a projectile of one form over its range of velocities and that for another projectile with a different form. The fact that a projectile in actual flight may gradually change its cross-sectional surface presented in the direction of flight owing to precession (and thus produce a gradual change in the "law") is an argument for making the measurements in a short distance.



30 cal. gov. 30 cal. 220 gr.

44 cal. 30 cal.

FIG. 5

Another ballistic application of the method of determining velocities is in connection with penetration. The energy loss, and even the law of retardation, that is to say the resistance of a solid with the passage of a body through it under given conditions, can be found through spark photography and successive determinations of the velocity. Here it is practically necessary to be able to measure the velocity in a few feet, this apparatus thus being particularly adapted.

While the ballistic application of the experiments is perhaps most important, yet the general investigation of the hydrodynamical aspects is of considerable interest. It is not necessary to restrict the bodies

which are hurled through a medium to those of practical ballistic design or dimensions. Coupled with the spark method of obtaining instantaneous photographs of the projectile and the waves connected with its flight, it is possible to examine the condition of the medium and the effect o change in body shape for a great range of conditions.

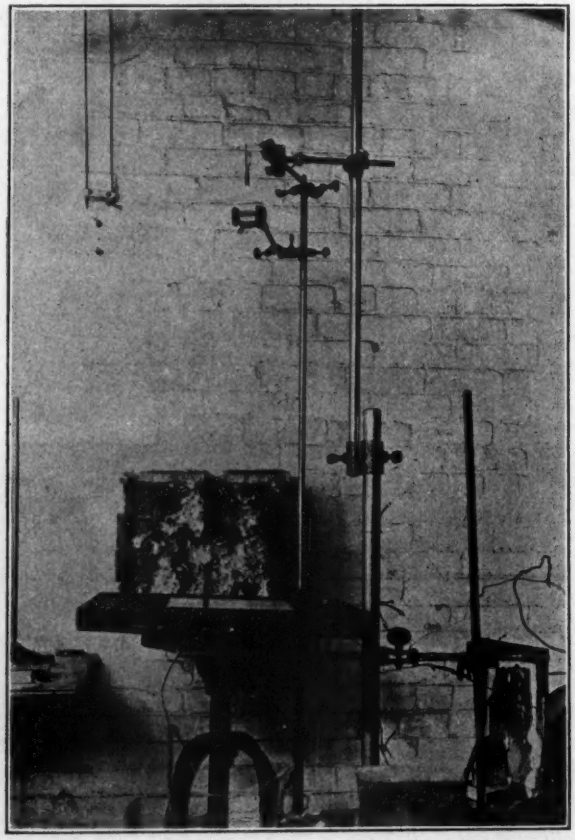


FIG. 6

The energy given up by a body in motion is dissipated in at least three ways—through skin friction, gas compression,[7] and production of the turbulent region particularly evident in the rear (see spark photographs, fig. 5). In order to formulate a theory of retardation it is necessary to have quantitative information regarding these factors and their relative importance. This information is very difficult to obtain.

Another application of the method of time calibration and of determining velocities comes in an entirely different field, namely, the measurement of the velocity of light. With a range of about 40 or 50 miles, which is available, a simple apparatus of this type will provide a record comparing the instant of arrival of light directly from the source (one of short duration, such as a spark or very rapidly moving slits) and that which has gone out and back over the 50 miles. The record will be similar to those which have been shown and obtained in much the same way. A precision which is at least as great as that obtained by the other methods which have been used for the measurement of the velocity of light is to be expected and in addition the permanent record, which will be given by a deflection on the film of the order of two centimeters, this representing the time taken for the light to travel the 50 miles.

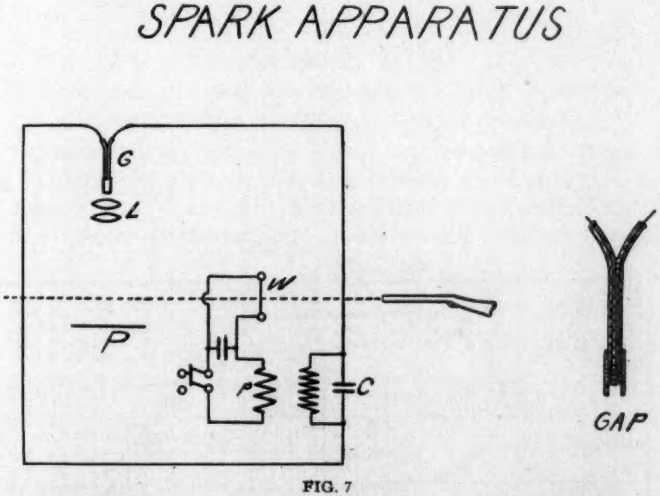


FIG. 7

The shadowgraphs (fig. 5) of projectiles in flight were made by use of the apparatus shown in figure 6, a diagram of which is shown in figure 7. The spark is obtained by breaking a small wire (w) through which the current passes to the primary of a large coil. The current is applied just before firing the gun, and in this way a very small wire can be used, not having time to fuse. The secondary condenser C controls the time interval elapsing between breaking the wire and the passing of the spark through the gap G which is constructed as shown in the figure. This makes a good "point" source and one always in the same place. The lens L is quartz. The position of the projectile on the plate can be controlled to perhaps 1 centimeter by moving the wire (w) with respect to the plate. This corresponds to a time interval of $^1/_{75,000}$ second.

[1] Contribution from the Ballistic Institute, Clark University, No. 7.

[2] Webster, A. G., "On A Means of Producing a Constant Angular Velocity," *Amer. Jour. Sci.*, **3**, May, 1897 (383–386).

[3] Webster, A. G., "An Experimental Determination of the Period of Electrical Oscillations," *Physic. Rev.*, **6**, 1898 (297–314); Hubbard, J. C., "An Experimental Determination of the Period of Electrical Oscillations," *Physic. Rev.*, **2**, 1, 1913 (247–249).

[4] Webster, A. G., *Proc. Nat. Acad. Sci.*, **5**, May 1919 (163–166).

[5] Webster, A. G. Thompson, L. T. E. "A New Instrument for Measuring Pressures in a Gun," *Proc. Nat. Acad. Sci.*, **5**, p. 258-263.

[6] Webster, A. G., "Some Considerations on the Ballistics of a Gun of Seventy-five-Mile Range," *Proc. Amer. Phil. Soc.*, **58**, p. 373–381. (Note by A. G. W. It should be stated that the linear law is not now considered exact. M. Lugot, of the Commission de Gâvre, informed the writer that the curves should rise a little above the straight lines, but the error is small.)

[7] Experiments to measure the compression are now in progress.—A. G. W.

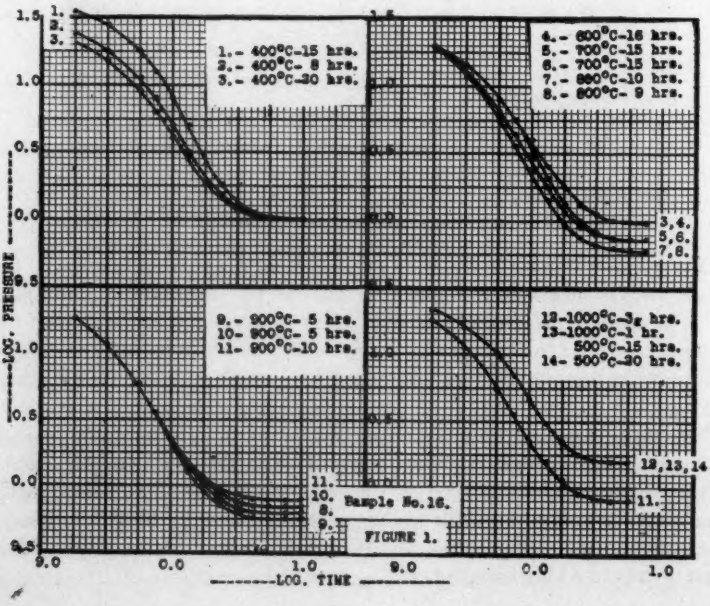
CHARCOAL ACTIVATION

By H. H. Sheldon

Ryerson Physical Laboratory, University of Chicago

Communicated by R. A. Millikan, February 20, 1920

The results obtained by Dr. Harvey B. Lemon on the variations due to heat treatment in the adsorption of gases by charcoal,[1] although published but recently, were obtained some time ago. There seemed two possible explanations of these results—either that the structure of the



charcoal was itself modified by heat alone or that the air was the agent causing these modifications. On this latter view a very complete theory, the hydrocarbon theory, has since been worked out by the Chemical Warfare Service.

The theory as published was not, however, substantiated by the appearance of any data in the journals, and accordingly certain experiments were begun here to satisfy ourselves regarding the conclusions arrived

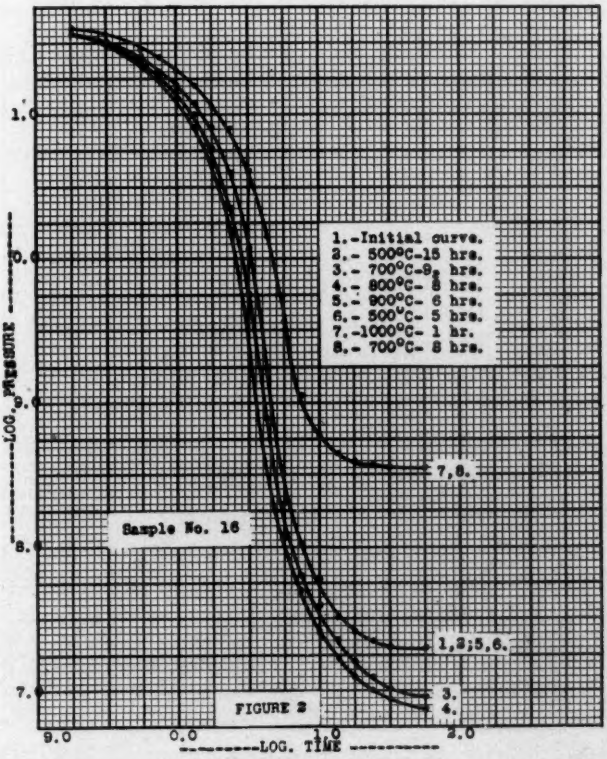


FIGURE 2

at. The apparatus and method were identical with those used by Dr. Lemon, and 25.7 gram of the same sample of charcoal, No. 16, was used. A similar sample to the one recorded was used as a check on all the work.

The constant end pressure after heat treatments up to 600° C. both in the hydrogen curves, figure 1, and in the nitrogen curves, figure 2, show that the activation recorded by Dr. Lemon for these temperatures, must have been due to oxidation. At temperatures between 600° C. and 900° C. activation takes place, accounted for by assuming that inactive hydrocarbons, stabilized by adsorption, are, however, being boiled away at this

temperature. Above this we find deactivation, due to the breaking up of hydrocarbons at this high temperature which form an inactive carbon deposit on the active base.

In the case of U. S. Government 600 minute charcoal, no such deactivation at this high temperature was observed, but in this charcoal the hydrocarbons are supposedly all removed. It offers no contradiction therefore.

The outgassings were as indicated on the next page.

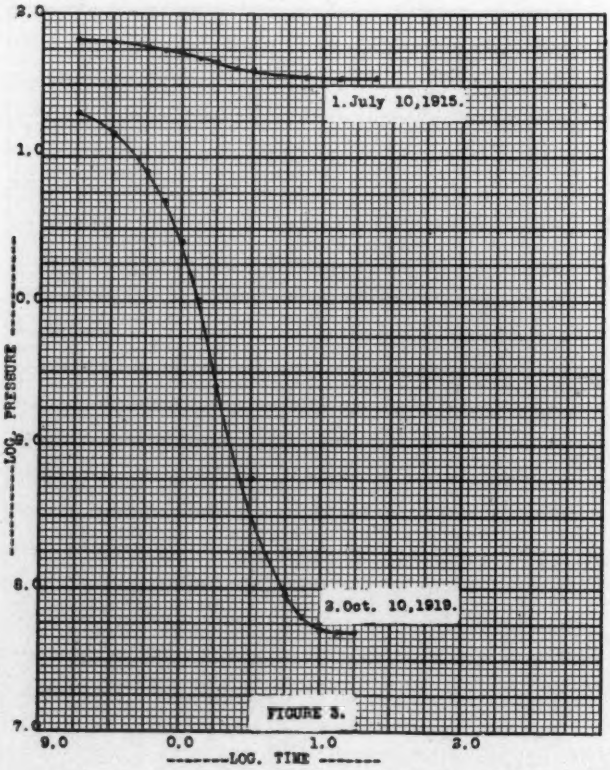


Figure 3 shows how charcoal may be activated by slow oxidation at room temperature. Curve 1 was taken July 10, 1915, and the sample was then put away and left undisturbed until Oct. 10, 1919, when curve 2 was taken.

The ease with which the charcoal could be deactivated for nitrogen compared to deactivation for hydrogen, suggested that a sample might be put into such a condition that it would adsorb hydrogen more readily than nitrogen. Results of this sort are shown in figure 4; curves 1 and 2 are nitrogen and hydrogen, respectively, before treatment, and curves

3 and 4, the same after treatment. This treatment consisted of $3^1/_2$ hours at 1000° C. in a vacuum, kept down by a condensation pump.

	Figure 1, Hydrogen			Figure 2, Nitrogen	
1	400° C.	15 hours	1	Initial Curve	
2	400° C.	8 hours	2	500° C.	15 hours
3	400° C.	20 hours	3	700° C.	$9^1/_2$ hours
4	600° C.	16 hours	4	800° C.	8 hours
5	700° C.	15 hours	5	900° C.	6 hours
6	700° C.	15 hours	6	500° C.	5 hours
7	800° C.	10 hours	7	1000° C.	1 hour
8	800° C.	9 hours	8	700° C.	8 hours
9	900° C.	5 hours			
10	900° C.	5 hours			
11	900° C.	10 hours			
12	1000° C.	$3^1/_2$ hours			
13	1000° C.	1 hour			
	500° C.	15 hours			
14	500° C.	20 hours			

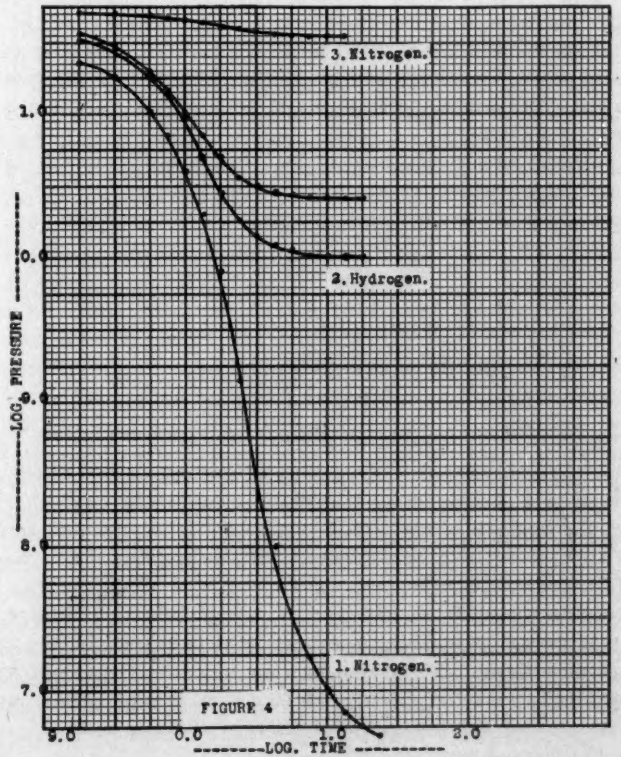


FIGURE 4

Several samples so treated worked successfully, and the fact that it was found impossible with U. S. Government 600 minute charcoal suggests that it was due to action of some sort between the hydrocarbons present and the gas. The fact that McBain's work[2] with hydrogen shows a dual action, and that work done later by others[3] on other gases points to a single action, suggests that while surface condensation is likely to explain the major action in both cases, that in the case of hydrogen there must be something else at work as well; either the hydrogen is dissolved or possibly combines with hydrocarbons unsaturated at this liquid air temperature. It is difficult to draw a distinct dividing line between the two, but the latter view is the one taken.

The deposit of inactive carbons on the active base, is according to this theory, much more effective in deactivating for nitrogen, since, interfering with surface condensation, it interferes with the whole effect, this not being the case for hydrogen which has a dual nature. The observations bear this out.

The decided drop in the initial end of the curves in figure 1 at the beginning of the work with hydrogen, which does not appear in the nitrogen curves of figure 2, again shows a difference in the adsorption of these two gases, possibly resulting from an increased fineness of the division of the material with which the hydrogen unites. Further work with other gases will be undertaken.

The author is greatly indebted to Dr. Harvey B. Lemon for valuable advice and suggestions in connection with this work.

[1] These PROCEEDINGS, **5**, July 1919, pp. 291–295.

[2] *Phil. Mag. London*, Ser. 6, **18**, 1909 (916).

[3] Miss Homfray, *Zs. Phys. Chem.*, **74**, 1910 (129).

THE STARTING OF A SHIP

BY JAMES K. WHITTEMORE

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Communicated by L. B. Mendel. Read before the Academy, November 11, 1919

In this paper we give first the results of a mathematical discussion of the motion of a particle under the action of tangential forces depending only on the velocity of the particle.[1] In the following paragraphs we suggest applications of these results to marine engineering and to the study of the law of resistance of liquids.

We consider a particle, P, moving in any path under the action of forces whose components along the tangent to the path depend on the velocity of the particle alone. We denote the time by t, the distance covered in the time t by x, the velocity at the start by v_0, at any time t by v, and the acceleration along the tangent by a. By hypothesis, $a = F(v)$. Concerning $F(v)$ we assume

$$F(v) = (\lambda - v)\varphi(v), \quad \lambda > v \geqq v_0,$$

and that the function $\varphi(v)$ is not less than a positive number m for such values of v and is finite for $v = \lambda$. It may then be proved

$$(\lambda - v_0)e^{-mt} \geqq \lambda - v > 0, \tag{1}$$

$$\lambda t - x = L - \int_v^\lambda \frac{dv}{\varphi(v)}, \tag{2}$$

$$L = \int_{v_0}^\lambda \frac{dv}{\varphi(v)}, \quad \frac{1}{m}(\lambda - v_0)e^{-mt} \geqq \int_v^\lambda \frac{dv}{\varphi(v)} > 0. \tag{3}$$

These results may be interpreted as follows: the velocity v increases from v_0 and approaches λ indefinitely as t increases without ever reaching that limit. But this "limiting velocity" λ is, for practical purposes, attained and is hereafter called "full speed." To interpret equation (2) we consider a particle P' describing the same path as P, starting from coincidence with P at the time $t = 0$, and moving uniformly with velocity λ; then evidently $PP' = \lambda t - x$. Equation (2) shows that PP' increases from zero approaching the finite limit L. We call L the "lost distance" or the distance lost in attaining full speed. It is clear that L is practically the distance that P is behind P' when P has reached full speed. If P is, for example, a ship attaining and then proceeding at full speed, L could be found from observation, for this lost distance is the difference of the distance run in any time τ at full speed and the distance run in the time τ from the start, provided that full speed is attained in the time τ from the start. We introduce the time T lost in attaining full speed, defined by $L = \lambda T$. This lost time T would, in the case of a ship or of a towed model, perhaps be more easily measured than L, for T is the difference of time in running any distance δ at full speed and in running the distance δ from the start, provided that full speed is attained in the distance δ from the start. Then if T is measured L is given by $L = \lambda T$. Now if the acceleration $F(v)$ is known L may be calculated from equation (3). If $F(v)$ is expressed by a formula containing an unknown constant and if L is found by observation the constant may be calculated from (3). These remarks lead to the suggestions of applications to the motion of a ship starting from rest or increasing speed under its own power or to the motion of a towed model.

For a towed model or for a ship increasing speed under the action of the thrust of the screw the component of accelerating force in the direction of the tangent to the path has the form, $f - r$, where f is the tension of the towing line in the case of a towed model or the effective thrust of the screw in the case of a ship, and r is the resistance of the water. No general expression for r is known, but it may be assumed that for any particular model or ship r is a function of v alone. We consider two forms of f: first f is constant, $f = k$; second f is the force exerted by a constant power, $f = c/v$.

Suppose $f = k$. Then if W is the weight of P,

$$\frac{W}{g}a = k - r(v), \quad a = K - R(v) = F(v).$$

Since $F(\lambda) = 0$, $K = R(\lambda)$, and

$$F(v) = R(\lambda) - R(v) = (\lambda - v)\varphi(v),$$

$$L = \int_{v_0}^{\lambda} \frac{dv}{\varphi(v)} = \int_{v_0}^{\lambda} \frac{(\lambda - v)dv}{R(\lambda) - R(v)}.$$

If we assume the resistance of the water proportional to some positive power of the velocity, $R(v) = K_1 v^n$, the conditions imposed on $F(v)$ are satisfied, and

$$L = \frac{1}{K_1} \int_{v_0}^{\lambda} \frac{(\lambda - v)dv}{\lambda^n - v^n}.$$

If in particular $v_0 = 0$ and if $n = 2$,

$$L = \frac{1}{K_1} \log_e 2 = \frac{0.693}{K_1}.$$

We have the curious result for this particular case that L is independent of λ and consequently of the force k. If L is obtained from observation K_1 is found from the preceding equations and the force required to develop speed λ is determined. Comparison of K_1 for different models would serve to determine the relative merits of their designs.

An apparatus might be simply constructed to give the effect of towing models with a constant force. Suppose, to outline such an apparatus in its simplest form, a model to be towed by a perfectly flexible horizontal cord passing over a pulley without weight or friction on its axis and attached to a descending weight. The tension of the cord would, under the assumptions, be the same on both sides of the pulley, but would vary with the velocity. The equations of motion for the model, weight W, and the descending weight W' are

$$\frac{W}{g}a = T - r(v), \quad \frac{W'}{g}a = W' - T,$$

where T is the tension. Adding the equations we have

$$a = \frac{g}{W + W'} [W' - r(v)],$$

from which it appears that the motion is the same as if the towing force were constant, the force of resistance however being modified by a constant factor. The preceding discussion would be applicable. In a more practical but less simple apparatus consisting of several cords and pulleys, the former not perfectly flexible, the latter neither weightless nor perfectly smooth, the results appear still to be applicable, account being taken of the stiffness of the cords and of the weight of the pulleys, but the fric-

tion of the pulleys on their axles must be eliminated or else the previous results cannot be applied.

If it is found possible so to run the engine of a ship as to deliver constant effective thrusting power at the screw we may give a similar discussion of the motion of the ship, where now $f = c/v$. Even if it is not possible to produce constant effective thrusting power in starting a ship from rest it may be found possible to do so in increasing speed and the following discussion would be applicable. We have

$$\frac{W}{g}\, a = \frac{c}{v} - r(v), \quad a = \frac{C}{v} - R(v) = F(v).$$

Since $F(\lambda) = 0$, $C = \lambda F(\lambda)$, and

$$F(v) = \frac{\lambda R(\lambda) - vR(v)}{v} = (\lambda - v)\varphi(v),$$

$$L = \int_{v_0}^{\lambda} \frac{dv}{\varphi(v)} = \int_{v_0}^{\lambda} \frac{(\lambda - v)v\,dv}{\lambda R(\lambda) - vR(v)}.$$

If $R(v) = K_1 v^n$ the conditions imposed on $F(v)$ are satisfied as before. If in particular $v_0 = 0$ and $n = 2$ we have $L = 0.247/K_1$, a value independent of λ and C. The assumption $R(v) = K_1 v^2$ gives $C = K_1 \lambda^3$, a formula connecting power and speed which is often used but which is certainly not correct.[2]

It is not possible to represent the resistance offered by the water to the motion of a model or a ship by an expression of the form $R(v) = K_1 v^n$. It is suggested that the law of resistance may be studied in two ways: more complicated laws of resistance may be assumed and tested by comparing the measured values of L with the values given by equation (3) under the assumed law. Either constant force or constant power may be used. Secondly, it may appear from experiment that the distance L for a model started from rest and brought to full speed λ, by a constant force, for example, can be expressed as a function of λ. Then the function $R(v)$ determining the law of resistance might be studied from the integral equation

$$L(\lambda) = \int_0^{\lambda} \frac{(\lambda - v)dv}{R(\lambda) - R(v)} = \lambda^2 \int_0^1 \frac{(1 - z)dz}{R(\lambda) - R(\lambda z)}, \quad v = \lambda z.$$

[1] The complete discussion is to be published in the *Annals of Mathematics*, probably in the number for June, 1920.

[2] See, for example, the "Admiralty Coefficient Formula," $H.P = \Delta^{2/3}V^3/K$, given by C. W. Dyson, *Practical Marine Engineering*, 7th edition, 1918, p. 614.

A THERMODYNAMIC STUDY OF ELECTROLYTIC SOLUTIONS

BY FRANK L. HITCHCOCK

DEPARTMENT OF MATHEMATICS, MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Communicated by Edwin B. Wilson, January 30, 1920

1. *Scope of the investigation.*—It is well known that aqueous solutions of strong electrolytes do not conform to the requirements of the usual mass law. Measurements of freezing-point and of electric conductivity unite in showing a marked rise of the "ionization-constant" with increase of concentration. The accumulation during recent years of a considerable body of accurate data renders a theoretical examination of these relationships highly desirable.

The theory of chemical potentials, due to Willard Gibbs, offers a ready tool for examining what the rigorous consequences of the dissociation hypothesis must be. From this theory may be deduced, for example:

1°. An expression for the heat of dilution as a function of temperature and of concentration.

2°. A rigorous equation for determining the freezing-point in terms of the concentration—nothing being assumed as to the "ideal" character of the solution.

3°. A generalization of the usual mass law.

These three equations will contain certain constants, dependent on the thermic properties of the solution. Some of these constants can be calculated from specific heats, or from latent heats of melting or of evaporation. Others, in the present state of our knowledge, must be determined empirically. The constants in the three equations, in so far as they are generalizations of the usual equations, are the same. Hence any results obtained under one head can be checked by the others.

2. *Heat developed by chemical or physical changes at constant temperature and pressure.*—It is well known that when a system at constant temperature and pressure undergoes any change, the heat Q emitted during the change is connected with the total free energy φ of the system by the differential equation

$$dQ = d(T\frac{\partial\varphi}{\partial T} - \varphi) \qquad (1)$$

This equation is quite general. It is not necessary that the change in question be reversible. If the system consists of several phases it is not necessary that the pressure be the same for all, provided it is constant for each.[1]

3. *The general condition of equilibrium.*—It is also well known that for a system of several phases whose composition is determined by masses $m_0, m_1, m_2, \ldots m_n$ the existence of a state of equilibrium implies the equation

$$\frac{\partial \varphi}{\partial m_0} dm_0 + \frac{\partial \varphi}{\partial m_1} dm_1 + \frac{\partial \varphi}{\partial m_2} dm_2 + \ldots + \frac{\partial \varphi}{\partial m_n} dm_n = 0 \quad (2)$$

which is one of the requirements that the free energy be a minimum. The partial derivatives $\partial\varphi/\partial m_0$, etc., were called by Gibbs the chemical potentials of the respective masses. I shall denote these chemical potentials by the symbols f_0, f_1, etc., that is

$$f_0 = \frac{\partial \varphi}{\partial m_0}, f_1 = \frac{\partial \varphi}{\partial m_1}, \ldots f_n = \frac{\partial \varphi}{\partial m_n} \quad (3)$$

so that the general condition of equilibrium may be written

$$f_0 dm_0 + f_1 dm_1 + f_2 dm_2 + \ldots + f_n dm_n = 0 \quad (4)$$

Equation (4) may be said to be the kernel of the Gibbs theory.

4. *Application to the melting-point curve of ice in contact with an aqueous solution.*—Let our chemical system now consist of m_0 mols of water in which are dissolved $m_1, m_2, \ldots m_n$ mols of various solutes in contact with a mass of ice m (without subscript). If any ice melts an equal amount of water is formed, that is $dm_0 = -dm$. The masses of the solutes are unchanged, that is $dm_1 = 0$, $dm_2 = 0$, etc. Hence, equation (4) becomes for this case

$$f_0 dm_0 - f dm_0 = 0$$

that is

$$f_0 - f = 0. \quad (5)$$

When ice is in equilibrium with an aqueous solution the chemical potential of the ice is equal to the chemical potential of the solvent.

It is important to notice that f_0 is not the potential of the solvent in the pure state, but its potential as it actually exists in the solution; it is a function of the concentrations of the various dissolved substances as well as of temperature and pressure. On the other hand f, the chemical potential of the ice, is a function of temperature and pressure only. It is also important to notice that one mol of ice or water here means merely one formula-weight, and assumes nothing as to actual molecular weight. Thus the chemical potentials are the free energies per formula-weight of their respective masses.

5. *The latent heat of melting ice in contact with the solution.*—When a little water freezes out of the solution the heat emitted is determined by equation (1). The only variables are m_0 and m. We may, therefore, write

$$dQ = \frac{\partial}{\partial m_0}\left(T\frac{\partial \varphi}{\partial T} - \varphi\right)dm_0 + \frac{\partial}{\partial m}\left(T\frac{\partial \varphi}{\partial T} - \varphi\right)dm \quad (6)$$

by the ordinary formula for a total differential. But we also have, identically,

$$\frac{\partial}{\partial m_0}\frac{\partial \varphi}{\partial T} = \frac{\partial}{\partial T}\frac{\partial \varphi}{\partial m_0} = \frac{\partial f_0}{\partial T} \text{ and } \frac{\partial}{\partial m}\frac{\partial \varphi}{\partial T} = \frac{\partial f}{\partial T} \quad (7)$$

while $dm_o = -dm$, so that (6) simplifies to

$$dQ = \left[T\frac{\partial f}{\partial T} - f - T\frac{\partial f_o}{\partial T} + f_o\right]dm \tag{8}$$

We may perfectly well suppose the total amount of solution to be so great that the freezing out of 1 mol of ice does not affect its composition. The expression in brackets is then a constant. Letting Q stand for the latent heat per mol we may, therefore, write

$$Q = T\frac{\partial f}{\partial T} - f - T\frac{\partial f_o}{\partial T} + f_o \tag{9}$$

6. *The form of the melting-point equation.*—If we adopt, as a temporary notation,

$$y = f_o - f \tag{10}$$

equation (9) becomes

$$-T\frac{\partial y}{\partial T} + y = Q \tag{11}$$

whose integral is, pressure being regarded as constant,

$$\frac{y}{T} = -\int\frac{Q}{T^2}dT + F(c) + K \tag{12}$$

where $F(c)$ is a function of the concentrations of the various solutes, but not of the temperature, and K depends on neither concentrations nor temperature. Since by (5) $y = 0$ when we have equilibrium, the equation of the melting-point curve takes the form

$$-\int\frac{Q}{T^2}dT + F(c) + K = 0 \tag{13}$$

7. *The chemical potential of the solvent.*—To determine the form of the function $F(c)$ we now proceed to get an expression for the potential f_o of the water in the solution. The concentrations may be defined as

$$c_1 = \frac{m_1}{m_o}, c_2 = \frac{m_2}{m_o}, \ldots c_n = \frac{m_n}{m_o} \tag{14}$$

Since the potential f_o is a continuous function of the concentration and approaches φ_o the potential of the pure solvent as the solution becomes infinitely dilute, we may, as a general concept, think of f_o as expanded in a power series in the concentrations, the first term being φ_o. It is very well known, however, that, as a first approximation, the diminution of potential when any solute is added to pure water is equal to RTc, that is, as a first approximation

$$f_o = \varphi_o - RT(c_1 + c_2 + \ldots + c_n) \tag{15}$$

which holds good at infinite dilution. As a second approximation we may assume that each of the dissolved substances acts like a perfect gas, which leads to the much more accurate equation[2]

$$f_o = \varphi_o - RT\ln(1 + c_1 + c_2 + \ldots + c_n) \tag{16}$$

which holds good when the solution is "ideal." It is evident that (15) is a limiting case of (16). If, as is the main object of the present paper, we wish to investigate solutions which are not ideal, we may write the perfectly general equation

$$f_o = \varphi_o - RT\ln(1 + \Sigma c) + P(c,T) \qquad (17)$$

where Σc means the sum of the concentrations of the various solutes, and $P(c,T)$ is a power series in concentrations and temperature. This series will, for the most general expression, contain the squares, products, and higher powers of the concentrations. It will not contain the first power, for (15) must be the limiting form of the equation as the solution becomes very dilute.

Instead of the power series $P(c,T)$ explicit algebraic functions might be introduced, based either on theoretical considerations or on empirical data.[3] The main point is that some form of function in concentration and temperature must be added to the right side of (16) to obtain sufficient generality. The object of this paper is to indicate how the entrance of such a function affects the ordinary theories of freezing-point, heat of dilution, and ionization, and for this purpose the power series appears the most convenient, and can be made as general as we please by taking terms enough.

It is important to notice that equations (15), (16) and (17) all assume that we know the molecular weights of the dissolved substances, that is, Σc depends on the actual number of dissolved molecules. This is fully in accord with the usual theory of very dilute solutions. But, equally important, nothing whatever is assumed as to possible hydration of these molecules, nor as to molecular complexes occurring in the solvent. As stated above, the number of mols m_o of solvent means merely formula-weight. It is precisely such matters as hydration, polymerization of solute, and departure from gas laws which are to be taken care of by the series $P(c,T)$.

It will be most convenient to expand this series first in powers of the absolute temperature T, thus

$$P(c,T) = P_o + P_1T + P_2T^2 + \text{etc.} \qquad (18)$$

It is doubtful whether we possess any data warranting the use of powers of T higher than the square. That the square, at least, is necessary will appear. The coefficients P_o, P_1 and P_2 will be power series in the concentrations only, it being assumed that the pressure is constant. (If we vary the pressure the coefficients in these new series will vary with pressure.) The expansion (18) is, however, general enough for our present purpose. We shall, therefore, have the potential f_o of the solvent in the form

$$f_o = \varphi_o - RT\ln(1 + \Sigma c) + P_o + P_1T + P_2T^2 \qquad (19)$$

where P_o, P_1 and P_2 are functions of the concentrations, which, if ex-

panded in series, will all begin with terms of the second degree. In other words, terms in the first powers of the concentrations occur nowhere except in the logarithmic term; if it were otherwise, the solution would fail to obey the perfectly well established laws for very dilute solutions.

8. *Determination of the function* $F(c)$.—Returning now to equation (12), writing $y = f_0 - f$, and putting for f_0 its value we have

$$\frac{1}{T}(\varphi_0 + P_0 - f) + P_1 - R\ln(1 + \Sigma c) + P_2 T = -\int \frac{Q}{T^2} dT + F(c) + K \quad (20)$$

where the only terms on the left which are functions of the concentrations alone are $P_1 - R\ln(1 + \Sigma c)$. Hence, by equating,

$$F(c) = P_1 - R\ln(1 + \Sigma c) \quad (21)$$

and by using this result the equation of the melting-point curve (13) becomes

$$-\int \frac{Q}{T^2} dT + P_1 - R\ln(1 + \Sigma c) + K = 0 \quad (22)$$

where, as already stated, P_1 is a function of concentration (or of pressure if that is not constant), but not of temperature.

9. *Heat of dilution.*—Before developing (22) further, it will be well to consider in a brief way the heat of dilution emitted when 1 mol of water is mixed with a large amount of solution under the same pressure. Take the complete system as consisting of a mass of pure water M together with the solution. The potential of pure water is φ_0. By reasoning precisely as in Art. 5, and letting H stand for the heat emitted when 1 mol of water is mixed with a large amount of solution,

$$H = T\frac{\partial f_0}{\partial T} - f_0 - T\frac{\partial \varphi_0}{\partial T} + \varphi_0 \quad (23)$$

Introducing the value of f_0 from (19) this becomes

$$H = P_0 - + P_2 T^2 \quad (24)$$

The absence of the term in the first power of T is noticeable.

If the heat of dilution at constant pressure be expanded in powers of T, *the first power is rigorously absent.*

9. *The equation of the melting-point curve.*—Returning now to (22), we may think of the heat of melting Q as composed of two parts, first the heat of melting at that temperature in contact with pure water, second the heat of dilution. The first we may call Q_0, the second is $-H$ (since Q was taken as heat emitted on freezing). Thus

$$Q = Q_0 - H \quad (25)$$

Without assuming the results of the last article, we may expand H in powers of T,

$$H = H_0 + H_1 T + H_2 T^2 \quad (26)$$

It is more convenient to expand Q_0 in powers of t, where $t = 0$ is the freezing-point of pure solvent,

$$Q_0 = q + q't \tag{27}$$

so that q is the latent heat of freezing per mol in contact with pure solvent, while q', to a good approximation, is the difference of specific heats of solvent in liquid and solid state. It is not likely that a term in t^2 will be needed with our present data, but can be introduced if wanted. It is important to notice that q and q' are not functions of concentration, by definition. Inserting values of H and Q_0 on the right side of the identity (20) and performing the integration we have (putting $t = T - T_0$),

$$\frac{1}{T}(\varphi_0 + P_0 - f) + P_1 - R\ln(1 + \Sigma c) + P_2T = \frac{q - q'T_0}{T} - \frac{H_0}{T} +$$

$$H_1\ln T + H_2T + P_1 - R\ln(1 + \Sigma c) + K - q'\ln T; \tag{28}$$

Comparing like terms on the two sides of this identity, we see that on the left the only term involving T^{-1} and varying with concentration is P_0/T while on the right the only like term is $-H_0/T$, hence $H_0 = -P_0$. Again, on the left there is no term in $\ln T$ varying with concentration, hence $H_1 = 0$. Terms in T give $H_2 = P_2$. Thus

$$H = -P_0 + P_2T^2$$

which checks with (24). Collecting results, and substituting in (22) we have the equation of the melting-point curve as

$$\frac{q - q'T_0}{T} + \frac{P_0}{T} + P_2T - R\ln(1 + \Sigma c) + K + P_1 - q'\ln T = 0 \tag{29}$$

where the constant K is yet undetermined. When $T = T_0$ the quantities P_0, P_1 and P_2 are zero since the concentration is zero. This gives for K

$$K = \frac{q}{T_0} + q' + q'\ln T \tag{30}$$

whence the melting-point equation by a little arrangement is

$$RT\ln(1 + \Sigma c) - P(c,T) = q\left[1 - \frac{T}{T_0}\right] - q'(T - T_0) - q'T\ln\frac{T}{T_0} \tag{31}$$

10. *Discussion of the melting point equation.*—Equation (31) departs from strict generality only in so far as we have failed to expand to higher powers of T. In principle it is entirely rigorous, and whenever we have data to warrant it we may expand as far as we please. It shows that a rigorous theory differs from the ordinary theory only through the function $P(c,T)$ which, if expanded in powers of the concentrations, begins with terms of the second degree, but which, if expanded in powers of T, contains a term P_1T playing no part in heat of dilution. To omit $P(c,T)$ altogether is, therefore, equivalent to something more than assuming heat of dilution to be zero.

If $P(c,T)$ be dropped, the equation becomes identical (except for slight difference of notation) with the melting-point equation established in a valuable paper by Washburn[4] on the two hypotheses:

1°. That the vapor of the solvent acts like a perfect gas.

2°. That Raoult's law holds good for the lowering of vapor pressure.

Since our equation (31) has been obtained without any recourse to vapor pressure, we may at once follow Washburn's reasoning in inverse order and get a certain amount of new light on the two assumptions about vapor pressure. For by differentiating (13) with respect to T we have

$$\frac{Q}{T_2} = F'(c)\frac{dc}{dT} \tag{32}$$

supposing there is only one dissolved substance. Using $F'(c)$ from (21) we have

$$\frac{Q}{T^2} = \left(P'_1 - \frac{R}{1+c}\right)\frac{dc}{dT} \tag{33}$$

which aside from slight difference of notation, differs from Washburn's differential equation in the presence of P'_1, which we have seen to be a function of c. Taking then the rigorous equation[5]

$$\frac{Q}{vT} = \left[\frac{\partial p}{\partial c}\right]_T \frac{dc}{dT} \tag{34}$$

where p is the vapor pressure, and eliminating dc/dT

$$\left[\frac{\partial p}{\partial c}\right]_T = \left(P'_1 - \frac{R}{1+c}\right)\frac{T}{v} \tag{35}$$

So far we have made no assumptions. If we assume $pv = RT$ we have by integrating

$$p_0 = p(1+c)e^{P_1} \tag{36}$$

a generalization of Raoult's law, to which it reduces if $P_1 = 0$. It is noteworthy that P_1 is the coefficient of precisely the term in the expansion of $P(c,T)$ which does not affect the heat of dilution. In other words the departure of the vapor from Raoult's law is dependent on the function $P(c,T)$, which affects the freezing-point, *minus* all the terms which affect the heat of dilution. Thus if heat of dilution and freezing-point are known we can find the vapor pressure, a fact also pointed out by Washburn. Equation (36) shows the precise relation of this fact to the melting-point equation provided we assume $pv = RT$. In any case (35) shows that the vapor pressure of a solution can be calculated if we know the coefficient of T in the chemical potential of the solvent, and the equation of state the vapor.

It will now be apparent that we can determine empirically the coefficients in the expansion of $P(c,T)$ and hence have a complete knowledge of the properties of the solution if we can get accurate determinations of heat of dilution at various temperature and over a considerable range of

concentrations, and also have accurate freezing-point determinations at fairly high concentrations, so as to calculate the missing term P_1. It is doubtful if any such data of high accuracy now exist over a wide enough range for even one substance. If, however, the accurate methods which have been used[6] to obtain freezing-point data for very dilute solutions can be carried to higher concentrations, and if the accurate data on heat of dilution[7] which have been obtained at 25° can be carried over a moderate temperature range, there seems to be no doubt we should have a theory of solution that would be practically complete. A detailed examination of existing data in the light of the foregoing theory is now under way, but would too greatly lengthen the present paper. A single example must here suffice. Taking cane sugar, and, as a first approximation, considering only terms in the square of the concentration, i.e., dropping the cube and higher powers in the expansion of $P(c,T)$ the coefficient C_2 of this term was calculated for values of N, the number of mols sugar per 1000 grams of water. The data[8] of Morse and Fraser gave

N.....	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2681
C_2.....	0.110	0.122	0.098	0.119	0.119	0.118	0.121	0.133	0.115

while the data[9] of Ewan gave

N.....	0.8714	1.2908	1.5347	1.6427	2.1108
C_2.....	0.118	0.104	0.132	0.128	0.132

Raoult's data[10] gave:

N.....	0.5056	1.0107
C_2.....	0.117	0.124

Other data quoted by Landolt and Bernstein gave

N.....	0.760	0.4236	0.2840
C_2.....	0.061	0.094	0.097

It is evident that the values of C_2 from the data of these different observers do not agree well. Below about 0.3 normal the solution departs so little from the "ideal" that the calculations become meaningless, depending as they do on differences between ideal and observed numbers.

11. *Application of the foregoing theory to electrolytes.*—Let us now suppose that the only dissolved substance is a single uni-univalent salt, as KCl. Calling c the formal concentration per 1000 grams of water we shall have

$$m_0 = 55.5,\ m_1 = m_2 = c\gamma,\ \text{and}\ m_3 = c(1 - \gamma)$$

where γ is the fraction of salt which is ionized, m_1 and m_2 the masses of the ions in mols, and m_3 the mols of un-ionized salt. At a given temperature $P(c,T)$ becomes a function of c alone and we may write

$$P(c) = Ac^2\gamma^2 + Bc^2\gamma(1 - \gamma) + Cc^2(1 - \gamma)^2 + \ldots \qquad (37)$$

where $A, B, C, \ldots$ are constants to be found. It is evident from the form of these terms that they will be without influence in very dilute solution,

hence ionization as calculated from freezing-point by the ordinary formula should be exact.

12. *Generalization of the mass law.*—The case is quite otherwise, however, when we consider equilibrium between the ions and un-ionized salt. For if we let f_1 and f_2 denote the chemical potentials of the ions, f_3 that of the un-ionized salt, the general condition (4) of equilibrium gives

$$f_1 + f_2 = f_3 \tag{38}$$

as the equation which determines the ionization. Now by definition

$$f_0 = \frac{\partial \varphi}{\partial m_0}, f_1 = \frac{\partial \varphi}{\varphi m_1}, f_2 = \frac{\partial \varphi}{\partial m_2} \text{ and } f_3 = \frac{\partial \varphi}{\partial m_4}$$

and if T be regarded as constant we may expand $P(c,T)$ in the form

$$P(c) = \frac{1}{m_0^2}[a_{111}m_1^2 + a_{112}m_1m_2 + \text{etc.}] + \frac{1}{m_0^3}[a_{1111}m_1^3 + a_{1112}m_1^2m_2 + \text{etc.}], \tag{39}$$

while, from (17), f_0 can be written

$$f_0 = \varphi_0 - RT \ln (m_0 + m_1 + m_2 + m_3) + RT \ln m_0 + P(c) \tag{40}$$

Taking the derivative with respect to m_1,

$$\frac{\partial f_0}{\partial m_1} = -\frac{RT}{m_0 + m_1 + m_2 + m_3} + \frac{1}{m_0^2}[2a_{11}m_1 + a_{12}m_2 + a_{13}m_3]$$

$$+ \frac{1}{m_0^3}[3a_{111}m_1^2 + 2a_{112}m_1m_2 + 2a_{113}m_1m_3 + a_{122}m_2^2 + a_{123}m_2m_3 + a_{133}m_3^2]$$

Now we have identically

$$\frac{\partial f_0}{\partial m_1} = \frac{\partial^2 \varphi}{\partial m_0 \partial m_1} = \frac{\partial f_1}{\partial m_0}$$

whence $f_1 = \int \frac{\partial f_0}{\partial m_1} dm_0 +$ terms not involving m_0. Therefore,

$$f_1 = \varphi_1 + RT \ln \left[\frac{m_1}{m_0 + m_1 + m_2 + m_3}\right]$$

$$- \frac{1}{m_0}[2a_{11}m_1 + a_{12}m_2 + a_{13}m_3] - \frac{1}{2m_0^2}[3a_{111}m_1^2, \text{etc.}]$$

where φ_1 involves temperature (and pressure) only. The presence of the term $RT \ln m_1$ is determined from our hypothesis that the solution acts like a perfect gas to a first approximation.[11] It is noteworthy that terms in f_0 involving *squares* of concentrations lead, in f_1, to terms of the *first* degree. Expressions for f_2 and f_3 are similarly found. If we put

$$x_1 = \frac{m_1}{m_0 + m_1 + m_2 + m_3}$$

with similar notation for x_2 and x_3, and substitute in (38) the expressions for f_1, f_2 and f_3 we shall have as the condition of equilibrium

$$\ln\left[\frac{x_1x_2}{x_3}\right] - G(c) = k \tag{41}$$

where $G(c)$ is a function of concentration, which, if expanded in series, begins with terms of the first degree in the masses of the dissolved substances. The coefficients in the expansion are functions of temperature (and pressure), and k is also a function of temperature. If we omit $P(c,T)$ in f_o we must also omit $G(c)$ in this extended mass law (41). It is noteworthy that in the logarithmic term we have rigorously x_1, x_2, x_3, not m_1/m_o, etc., although in solutions not very concentrated it does not matter much.

13. *Discussion of the generalised mass law.*—It is shown in the foregoing theory that if the function $P(c,T)$ can be accurately determined we can at once calculate the function $G(c)$ and hence predict to what extent ionization departs from the mass law. This could be done from accurate measurement of heat of dilution, checked as to terms in T by freezing-point data. Conversely, if ionization be known, we can find $G(c)$ and hence $P(c)$ for that temperature. Many other relations can be brought to bear particularly those connecting specific heats with change in equilibrium under varying temperature.

Furthermore equation (41) can be employed to study the theoretical form of the curve obtained by plotting the "constant" K against the formal concentration c. By putting $m_1 = m_2 = c\gamma$ and $m_3 = c(1 - \gamma)$ we may write (41) as

$$K = \frac{\gamma^2 c}{1 - \gamma} = K_o e^g \tag{42}$$

where g is a function of concentrations which may be expanded

$$g = ac\gamma + a'c(1 - \gamma) + \text{etc.} \tag{43}$$

By differentiating with respect to c we have

$$\frac{dK}{dc} = K_o e^g \frac{dg}{dc} \tag{44}$$

while dg/dc found from (43) is

$$\frac{dg}{dc} = a\gamma + a'(1 - \gamma) + (a - a')c\frac{d\gamma}{dc} \tag{45}$$

and the value of $d\gamma/dc$ from (42) is

$$\frac{d\gamma}{dc} = \frac{[ac\gamma + a'c(1 - \gamma) - 1]\gamma(1 - \gamma)}{c[2 - \gamma + c\gamma(1 - \gamma)(a' - a)]} \tag{46}$$

Now as c approaches zero, γ approaches unity. By inspection of (46) we see that $cd\gamma/dc$ approaches zero, for the denominator approaches unity on cancelling c. Hence from (45) we see that dg/dc approaches the constant a. Hence by (44) we have

$$\text{Lim}\,\frac{dK}{dc} = aK_0 \qquad (47)$$

This means that we shall not naturally expect the electrolyte to obey the mass law even in very dilute solution. For this would, in strict rigor, mean that the graph of K against c should have a horizontal tangent as c approaches zero. However, a curve differing little from the horizontal would be hard to distinguish experimentally from the other.

In a highly interesting investigation Washburn has worked out this curve for KCl at extreme dilutions,[12] assuming that the graph of K shows no marked change in character as c approaches zero. To compare his results with the present theory I have assumed that his value of Λ_0 is correct and also his value 0.02 for K_0. Determining the function g so that the following data (from Washburn and from Adams) should be exactly satisfied:

c.....	0.0004	0.001	0.005
γ.....	0.98923	0.98163	0.958

the function g was found to satisfy

$$g \log_{10} e = 1075c\gamma - 47234c(1 - \gamma) + 240064c^2\gamma^2$$

Values of γ calculated from this equation were then compared with Washburn's results for round concentrations:

c.....	0.00005	0.0001	0.0002	0.0003	0.0007
γ.....	0.99796	0.99595	0.9931	0.9910	0.98507 (Calc.)
γ.....	0.99753	0.99529	0.99256	0.99083	0.98511 (Washburn)

The curve for K plotted from the calculated values of γ lies very slightly higher than Washburn's curve at concentrations below 0.0004, but approaches 0.02 as c approaches zero. It coincides almost exactly with Washburn's curve above 0.0004 differing from his curve in having but slight inflection. An inspection of his diagram makes it clear that a curve meeting the vertical axis at a small angle with the horizontal, and having but little inflection still fits his data very well, and, of course, would fit entirely with his supposition that no marked change occurs in the character of the curve at infinite dilution. It is highly probable, too, that the empirical value of g given above is not the best that can be found. Further investigation is now being conducted.

14. *Summary.*—The adoption of Gibbs' principle of chemical potentials leads to an extention of the ordinary theories of melting-point, heat of dilution, vapor pressure, and mass law. In all cases these extensions are the result of the presence, in the expression for the chemical potential of the solvent, of terms in the square and higher powers of the concentrations. In many phenomena these terms are without measurable effect in dilute solution. They lead in the case of the mass law to terms in the first

power of the concentrations, hence are important even in very dilute solutions.

[1] Duhem, *Méchanique Chimique*, Livre VI, Chap. 111. The function $T\partial\varphi/\partial T - \varphi$ is identically equal to Gibbs' heat function $U + pv$.

[2] *Ibid.*

[3] Van Laar, *Sechs Vortrage u. d. thermodynamische Potential*, S. 84.

[4] *Technology Quart., Boston*, **21**, 1908 (372).

[5] *Ibid.*, p. 371.

[6] Adams, L. H., *J. Amer. Chem. Soc.*, **37**, 1915 (481).

[7] MacInnes and Braham, *Ibid.*, **39**, 1917 (2110).

[8] *Amer. Chem. Jour.*, **34**, 1905; **37, 38**, 1907.

[9] *Zs. phys. Chem.*, **31**, 1899; and **27**, 1898.

[10] Duhem, *loc. cit.*

[11] These PROCEEDINGS, **3**, 1917 (569); see also *J. Amer. Chem. Soc.*, **40**, 1918 (106–158).

[12] These PROCEEDINGS, **3**, 1917 (574) and *J. Amer. Chem. Soc.*, **40**, 1918 (145).

THE FORMATION OF THE CELL PLATE IN THE CAMBIUM OF THE HIGHER PLANTS

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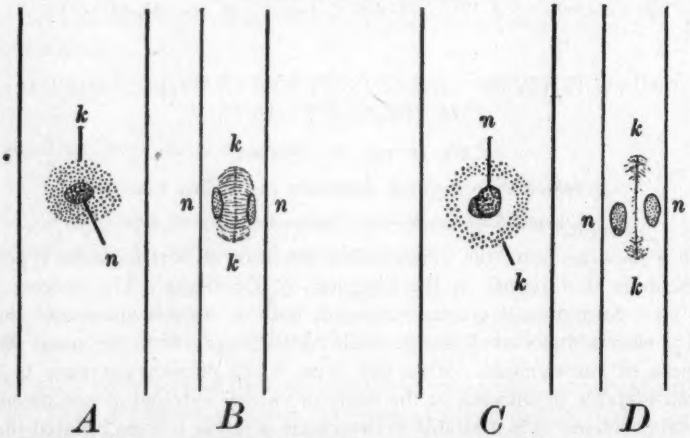
In a previous note[1] the writer called attention to a remarkable type of cytokinesis that occurs in the cambium of Coniferae. The process of cell plate formation is greatly extended, both as regards space and time and is clearly dissociated, except in its initial stages, from the usual phenomena of karyokinesis. Since this type of cell division promises to be of considerable significance in the study of various cytological and physiological problems[2] it is desirable to determine whether it is an isolated phenomenon, i.e., confined to the Coniferae, or one that is characteristic of the cambia of all of the higher plants, angiosperms as well as gymnosperms.

During the last growing season, I secured specimens of the cambium from an extensive series of angiosperms. Selections were made so as to include representatives of all of the larger and more important orders of the dicotyledons and of certain arborescent monocotyledons which have "secondary" growth in thickness. Specimens were obtained from both tropical and temperate environments.

Although the cambial initials in mature stems of angiosperms are on an average considerably smaller than homologous elements of gymnosperms[3] the salient features of cytokinesis are the same in both subphyla. In the former, as in the latter group, each initial contains a single nucleus which is centrally located and divides mitotically. The spindle becomes extended laterally by the addition of peripheral "fibers"[4] and gradually assumes the form of a disk, figures *A* and *B*. As more fibers are successively added the original "connecting fibers" disappear from about the cell

plate, leaving a ring-shaped aggregation of kinoplasmic fibers, which in tangential longitudinal sections of the cambium appears as a halo about the daughter nuclei, figure *C*. This ring increases in circumference until it intersects the radial walls, when it becomes more or less flattened on two sides. As soon as the cell plate intersects the radial walls of the cell, the fibers in these two sides disappear leaving two separate aggregations of kinoplasmic fibers which are connected by the cell plate, figures *E*, *F*, *G*, *H*. These aggregations of fibers, kinoplasmasomes, have the same characteristic shape and structure as those which occur in the Coniferae. They extend across the cell—more or less at right angles to its longitudinal axis—from one radial wall to the other, figure *G*, and are

Robinia Pseudo-Acacia L.



A. Central portion of cambial initial, in tangential longitudinal extension, showing early "disk" stage in the formation of the cell plate. X 900.

B. The same. Radial longitudinal extension. X 900.

C. Central portion of cambial initial, in tangential longitudinal extension, showing "ring" or "halo" stage in the formation of the cell plate. X 900.

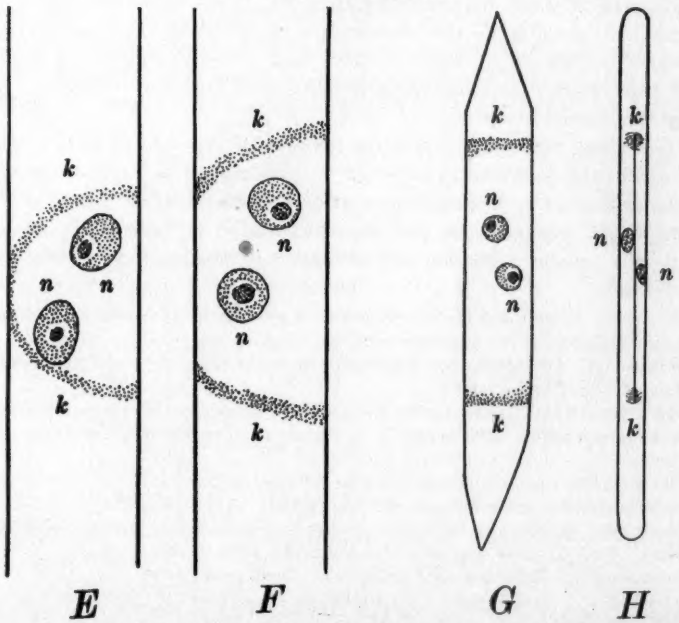
D. The same. Radial longitudinal extension. X 900.

located in the center of the protoplast, midway between its tangential surfaces, figure *H*. They move in opposite directions—*usually at equal rates*—towards the ends of the cell. As they move forward, the cell plate is extended until it eventually reaches the two ends of the protoplast, thus dividing it into halves, each of which contains one of the daughter nuclei. The latter usually remain close together near the center of the cell during cytokinesis, as in the gymnosperms; and I have been unable to find any visible connection between them and the kinoplasmasomes.

Having determined that this type of cytokinesis is characteristic of the cambia of both gymnosperms and angiosperms, it is of interest to in-

quire whether it ever occurs in other somatic tissues, and what relation it bears to those types of cell plate formation which previously have been described by Treub,[5] Strasburger[6] and Schürhoff.[7] The writer has accumulated considerable evidence which indicates that the phenomenon in question is not confined to the cambium, but occurs in other somatic tissues, in elongated or much flattened cells whose planes of division have

Robinia Pseudo-Acacia L.



E. Central portion of cambial initial, in tangential longitudinal extension, showing disappearance of two arcs of the kinoplasmic ring. X 900.

F. Central portion of cambial initial, in tangential longitudinal extension, showing early kinoplasmasome stage in the formation of the cell plate. X 900.

G. Cambial initial, in tangential longitudinal extension, showing later kinoplasmasome stage in the formation or the cell plate. X 450.

H. The same. Radial longitudinal extension. X 450.

n. Daughter nucleus.

k. Kinoplasma.

one long and one short dimension. Furthermore, a comparative study of cytokinesis in different somatic tissues and in cells of different shapes and sizes suggests that the various types of cell plate formation, described by Treub, Strasburger, Schürhoff and the writer, are but different phases or stages of a single general or fundamental type of cytokinesis. The particular expressions of the phenomenon which may occur in any given

cell are dependent upon the dimensions of the latter, its plane of division and the size and location of the nucleus. Thus, in very small, isodiametric cells, having a large centrally located nucleus, the cell plate quickly intersects the walls of the cell, without any extensive laterial growth—by means of additional peripheral fibers—of the spindle. In somewhat larger elements there is sufficient room for the process of cytokinesis to reach the "halo" stage before the cell plate intersects the sides of the cell. Only in elongated or much flattened elements, i.e., cells which are not isodiametric, is it possible for the phenomenon of cell plate formation to pass through the "spindle," "disk," "halo" and "frame" stages and finally form two entirely separate aggregations of kinoplasmic fibrillae, the kinoplasmasomes.

It is evident, accordingly, that the remarkable type of cell plate formation which was described in a former note, is not an isolated or unusual phenomenon, but is of frequent occurrence in the somatic tissue of the higher plants, gymnosperms and angiosperms. It promises to be significant in any general discussion concerning the dynamics of cytokinesis and karyokinesis.

[1] Phenomena of cell division in the cambium of arborescent gymnosperms and their cytological significance, these PROCEEDINGS, **5**, 1919 (283–285).

[2] Bailey, I. W., The significance of the cambium in the study of certain physiological problems, *J. Gen. Physiol.* Ined.

[3] The average length of the cambial initials in 152 stems of gymnosperms was computed as approximately 3400 micra; in 275 stems of dicotyledons as approximately 600 micra.

[4] The word fibers is used without reference to whether the fibrillae are true threads or lines of protoplasmic streaming in the living cell.

[5] Treub, M., "Quelques recherches sur le rôle du noyau dans la division des cellules végétales," *Verh. K. Akad. Wetensch. Amsterdam*, **19**, 1879 (1–35).

[6] Strasburger, E., *Zellbildung und Zelltheilung*, 3 aufl, Jena, 1880.

[7] Schürhoff, P., "Das Verhalten des Kernes im Wundgewebe," *Beih. Bot. Central.*, **19**, 1906 (359–382).

FUNCTIONALS INVARIANT UNDER ONE-PARAMETER CONTINUOUS GROUPS OF TRANSFORMATIONS IN THE SPACE OF CONTINUOUS FUNCTIONS

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Communicated by G. A. Bliss, February 20, 1920

1. *Some general notions.*—It is intended in this abstract to give a few examples of one-parameter continuous groups of transformations in function space which are analogues of well-known groups in the space of a finite number of dimensions, and to exhibit in each case a functional invariant in terms of which every invariant of the group (with suitable restrictions) is expressible. Some of these groups have already been con-

sidered by G. Kowalewski,[1] although, so far as the writer knows, no attempt has been made to discover the invariants of the group.

Let u or $u(\xi)$ be a real continuous function defined on the interval $a \leqq \xi \leqq b$, and τ a real variable ranging over the interval $|\tau - \tau_0| \leqq c$. Consider also a functional operation $F[x,u,z]$ which, for every element (x, u, τ) of the set defined by

$$a \leqq x \leqq b, \quad \max_{\xi} |u(\xi) - u_0(\xi)| \leqq \alpha, \quad |\tau - \tau_0| \leqq c$$

determines a real number. If, furthermore, F is a continuous functional[2] of its arguments, then for each τ, one may regard F as a transformation taking the continuous function u into another continuous function $\bar{u}$ of the variable x,

$$\bar{u}(x) = F[x,u,\tau] \tag{1}$$

As usual, the transformations (1) are said to form a group if the product of every two transformations of the set is in the set, and if, furthermore, the set contains the identity transformation and for every transformation the corresponding inverse transformation.

An infinitesimal transformation is defined by the equation

$$\frac{\partial u(x,\tau)}{\partial \tau} = f[x,u], \tag{2}$$

where

$$f[x,u] = \frac{\partial}{\partial \tau} F[x,u,\tau]_{\tau = \tau_0}$$

τ_0 denoting the parameter giving the identity transformation of (1). If one supposes that the functionals have suitable properties of continuity and differentiability,[2] one can show that equation (2) determines a one-parameter continuous group of transformations.

By a method exactly like that employed by Lie it can be shown that the functionals $G(u)$ which are finite invariants (*finite* is here used in contradistinction from *differential*) of the group (1) may be found from the solutions of the Stieltjes integral equation

$$\int_a^b f[x,u] d_x \varphi[x,u] = 0 \tag{3}$$

where the φ is defined by

$$\int_0^1 u(x) d_x \varphi[x,u] = \Phi[u],$$

and Φ is the Frêchet differential[3] of G. It can be shown that if the functional f has the properties already referred to, solutions of (3) exist.[4]

2. *Examples.*—The set of transformations

$$\bar{u}(x) = u(x) + \alpha(x)\tau = F[x,u,\tau]$$

where $\alpha(x)$ is an arbitrary but specified continuous function evidently

represents a group in function space with $\tau = 0$ as the identity parameter. The infinitesimal transformation is given by the equation

$$\frac{\partial u(x,\tau)}{\partial \tau} = \alpha(x).$$

Furthermore, every invariant satisfying suitable restrictions of continuity and differentiability may be expressed as a functional of the invariant

$$I[x,u] = \begin{vmatrix} u(x) & \int_a^b u(\xi)d\xi \\ \alpha(x) & \int_a^b \alpha(\xi)d\xi \end{vmatrix}.$$

The transformations

$$\bar{u}(x) = \tau u(x) = F[x,u,\tau]$$

form a group. The infinitesimal transformation is

$$\frac{\partial u(x,\tau)}{\partial \tau} = u(x),$$

and every invariant is a functional of[5]

$$I[x,u] = \frac{u(x)}{\int_a^b u(\xi)d\xi}.$$

It is easily verified that the transformations

$$\bar{u}(x) = u(x)e^{-\tau} + (e^{\tau} - e^{-\tau})\int_0^1 u(\xi)d\xi = F[x,u,\tau]$$

form a group with $\tau = 0$ giving the identity transformation. The infinitesimal transformation is defined by the integro-differential equation

$$\frac{\partial u(x,\tau)}{\partial \tau} = -u(x) + 2\int_0^1 u(\xi)d\xi.$$

Every invariant is a functional of

$$I[x,u] = u(x)\int_0^1 u(\xi)d\xi - \left\{\int_0^1 u(\xi)d\xi\right\}^2.$$

Consider the group defined by the infinitesimal transformation

$$\frac{\partial u(x,\tau)}{\partial \tau} = \int_0^{2\pi} \sin \sigma(x - \xi)u(\xi)d\xi$$

where σ is an odd integer. It can be shown[6] that the finite group may be put in the form

$$\bar{u}(\quad = u(x) + \frac{1}{2\pi}\Big[e^{-\sigma ix}(e^{-\sigma i\tau} - 1)\int_0^{2\pi} e^{\sigma i\xi}u(\xi)d\xi + e^{\sigma ix}(e^{\sigma i\tau} - 1)\int_0^{2\pi} e^{-\sigma i\xi}u(\xi)d\xi\Big] = F[x,u,\tau]$$

Every invariant is a functional of the invariant

$$I[x,u] = u(x) + \frac{1}{2\pi}\left[1 - \int_0^{2\pi} e^{\sigma i\xi}u(\xi)d\xi\right]\left[e^{-\sigma ix} - e^{\sigma ix}\int_0^{2\pi} e^{-\sigma i\xi}u(\xi)d\xi\right].$$

Some interesting special invariants are obtained when the particular invariant functionals

$$\int_0^{2\pi} e^{-\sigma ix}I[x,u]dx \text{ and } \int_0^{2\pi}\Big\{I[x,u]\Big\}^2dx$$

are taken in which cases one obtains, respectively, the invariants

$$\left[\int_0^{2\pi}\cos\sigma\xi u(\xi)d\xi\right]^2 + \left[\int_0^{2\pi}\sin\sigma\xi\, u(\xi)d\xi\right]^2, \int_0^{2\pi} u^2(x)dx.$$

This group is a special instance of the orthogonal group in function space[1] defined by the infinitesimal transformation

$$\frac{\partial u(x,\tau)}{\partial\tau} = \int_a^b K(x,\xi)u(\xi)d\xi,$$

where $K(x,\xi)$ is a skew-symmetric kernel; i.e., $K(x,\xi) = -K(\xi,x)$.[3] The invariants of this group are quite complicated and will not be given.

As a last example, let a group have the last mentioned infinitesimal transformation where the given kernel function $K(x,\xi)$ is continuous and symmetric. The finite transformations may be put in the form[6]

$$\bar{u}(x) = u(x) + \sum_{n=1}^{\infty}\varphi_n(x)(e^{\frac{\tau}{\lambda_n}} - 1)\int_a^b \varphi_n(\xi)u(\xi)d\xi = F[x,u,\tau],$$

where the constants λ_n are the characteristic numbers of the kernel $K(x,\xi)$ and the functions $\varphi_n(\xi)$, are the corresponding normed orthogonal set of characteristic functions. Every invariant is a functional of

$$I[x,u] = u(x) + \sum_{n=1}^{\infty}\varphi_n(x)\left\{\left[\int_a^b \varphi_n(\xi)u(\xi)d\xi\right]^{\frac{\lambda_1}{\lambda_n}}\frac{1 - \left[\int_a^b \varphi_1(\xi)u(\xi)d\xi\right]^{\frac{\lambda_1}{\lambda_n}}}{\left[\int_a^b \varphi_1(\xi)u(\xi)d\xi\right]^{\frac{\lambda_1}{\lambda_n}}}\right\}.$$

If the particular invariant functional

$$\int_a^b I[x,u]\varphi_k(x)dx, \quad k \neq 1,$$

is taken, one obtains the invariants

$$\frac{\int_a^b \varphi_k(\xi)u(\xi)d\xi,}{\left[\int_0^1 \varphi_1(\xi)u(\xi)d\xi\right]^{\frac{\lambda_1}{\lambda_k}}}, k \neq 1.$$

One could now try to extend some other results of the theory of continuous groups to functions both for one parameter and for several parameters. For example, it would be interesting to know what the invariant manifolds of a given group are; also the applications to partial differential equations in infinitely many variables, which the writer is now engaged

in investigating. Finally, groups in which the functions $u(x)$ are discontinuous would seem to be worthy of consideration.

[1] G. Kowalewski, *Sitz. ber. Math. Akad. Wiss. Wien*, **120**, Abteilung IIa[1], IIa[2], 1911 (77–109,1435–72).

[2] See writer's doctor's dissertation (Chicago, 1918).

[3] *Trans. Amer. Math. Soc.*, **15**, 1914*r* (139).

[4] In a paper by the writer soon to be published on Linear Partial Differential Equations with a Continuous Infinitude of Variables.

[5] This formula has already been obtained by Volterra as the general solution of a functional differential equation in his paper, *Atte R. Accad. Lincei*, Ser. 6, **23**, 1914, 1st semester, (p. 393.)

[6] See paper by writer entitled, Integro-differential Equations with Constant Kernels, *Bull. Amer. Math. Soc.*, **26**, 1920 (193).

THERMOKINETICS OF LIOMETOPUM APICULATUM MAYR

BY HARLOW SHAPLEY

MOUNT WILSON OBSERVATORY, PASADENA, CALIFORNIA

Communicated by W. M. Wheeler, February 16, 1920

Variation in the activity of a cold-blooded animal is largely dependent on metabolic changes, which in turn probably depend mainly on the accelleration of oxidation and of other chemical reactions. The physical nature of the environment affects most of these chemical processes, and we should expect that the same physical properties would also directly influence the kinetic manifestations of animal life. While many observations record the qualitative relation of animal activity to such factors as humidity and external temperature, only a few give definitive numerical results.

An opportunity for precise quantitative measurement of the variation in the rate of expenditure of animal energy under the different natural stimuli arising from changing meteorological conditions, is afforded by the study of the foraging and patrolling activities of ants of the subfamily Dolichoderinae. Although some of my observations have been made on other genera of this group (*Tapinoma*, *Dorymyrmex*), I find the most suitable material to be the "Argentine ant," *Iridomyrmex humilis* Mayr, and two species of *Liometopum*. In briefly describing the activities of these ants and the observations based upon them, a number of reasons are proposed in the following paragraphs why members of the subfamily, and particularly the California *Liometopa*, are thought to be better adapted than nearly any other organism for the close quantitative investigation of kinetic response to changes in field conditions.

The trail-running habit, which is common to all genera of Dolichoderinae except *Leptomyrmex* of Australia,[1] is fundamental for the observations discussed below. It permits permanent observing stations to be set up along the trail so that the speed over fixed intervals of distance can be

measured under different conditions of light, air-temperature, relative humidity, wind-velocity, and barometric pressure, and at different times of the day and during all seasons of the year. The trails are maintained for long intervals of time, many of those of *Liometopum apiculatum* and *L. occidentale* on Mount Wilson have not changed during two years of observation. Colonies of the closely related *L. microcephalum* of Europe are also known to maintain constant trails for several years.[2] Ants running in these files, whether toward or away from the nest, are for the most part unburdened, their function apparently being mainly that of patrol.

Another characteristic of the *Liometopa*, which is very fortunate for this research, is that they are equally diurnal and nocturnal in habit—a characteristic that permits continuous series of observations as well as favorable opportunities for the investigation of natural phototactic reactions.

The conspicuous differences in stature of the workers of *Liometopum* allow the possible correlation of activity and hardihood with size. The average live weight of a worker maxima of *L. apiculatum* is, according to my measures, about five hundredths of a grain (140,000 to a pound); the average worker minor weighs half as much, but the extremes of weight for maxima and minima are probably in the ratio of four to one.

The wide range of temperature, humidity, wind, and light, throughout which these ants are active, enables a thorough testing of the extent to which the various meteorological factors are involved in kinetic reactions. The files are maintained with the relative humidity near its maximum of 100%, or when as low as 5%; the patrols are running when the temperature is 38° centigrade (over 100° Fahrenheit) and also when it is less than 8°. After a few warm days late in December thousands of ants of two species of *Liometopum*, on Mount Wilson, were out of their nests only a few feet from banks of snow.

There is a distinct advantage for thermokinetic studies in the comparative freedom of the patrolling *Liometopa* from the erratic motions which are characteristic of many kinds of insects, or which are in fact characteristic of most active animals of all the higher phyla. It is this common irregularity and discontinuity of action that usually prevents quantitative measurement of the kinetic responses of cold-blooded animals. A fixed patrol of Dolichoderines from a populous colony, however, is essentially a steady flow of animal energy; its quantity is governed largely by food and its speed by meteorological conditions. Except at low temperatures the normal speed of running appears to be very near the maximum speed possible under the prevailing conditions. For temperatures below 15° centigrade the excitation of battle or food temporarily increases activity.

The California *Liometopa* are carnivorous, granivorous, and aphidicolous. This catholicity of taste permits successful baiting for the purpose of main-

taining the density of a file, for the establishment of new trails over some special terrain, or for other experiments.

The large population of the nests makes possible the establishment of files of sufficient density for statistical treatment of the observations of speed. The population of colonies of *Liometopum* and of *Iridomyrmex humilis* is to be numbered by the tens of thousands. On an average patrol during the warmer months, 100 ants pass a given point in one minute (see table III, p. 209). From one nest on Mount Wilson, in the summer of 1919, four files issued, and every day, under favorable conditions of weather, 70,000 ants went out along each file and nearly 70,000 ants came in.

The observations tabulated below were made on Mount Wilson (altitude 5700 feet) on ants from a nest of *L. apiculatum*, located in the basal, slightly-decayed trunks of a group of *Quercus chrysolepis* Liebm. This species of *Liometopum* is not assigned to the state in Wheeler's "Mountain Ants of Western North America."[3] The subspecies *luctuosum* Wheeler, however, has been taken on nearby mountains[4] and probably occurs on Mount Wilson. In a discussion of California ants, Woodworth[5] states that "Liometopum is a very rare insect," it is probably uncommon, therefore, in many parts of the state, but in the San Gabriel mountains it is by far the most numerous ant, with the species *occidentale* Emery predominating.[6]

Of the nine different "speed traps" set up for the study of the nest of *L. apiculatum*, Stations A and Y were principally used. The former is at the base of a concrete wall some 50 feet from the nest; the latter, on another trail, is on a plank surface about 2 feet from the main nest openings. These two pieces of trail are of comparable smoothness, and each is 30 centimeters in length. The time required for individuals to pass through the speed traps was recorded (with a stop watch for high temperatures), together with an indication of the size of ant and its direction of travel. At the beginning and end of each observation, which is the mean time in seconds for about 20 individuals, the temperature was read from a thermometer, the bulb of which was placed against the wall or board, some 10 or 15 millimeters from the trail.

As the observations are made within a few yards of a special station maintained by Mr. Hoge for the United States Weather Bureau, we have available, for all the observations on speed, continuous graphical records of the precipitation, wind-velocity, temperature, and barometric pressure, as well as semi-daily cloud and psychrometric observations.

It was found from the first observations that the kinetic response for these ants is essentially a function of temperature alone—other meteorological conditions (except rain, of course) having very little if any direct effect on speed. Even the influence of light and time of day are extremely small compared with the influence of temperature. If the speed is de-

pendent mainly on the velocity of chemical reactions, this result should be expected.

The quantities in the sixth column of table I are the observed mean time required for an ant to travel 30 centimeters, at the corresponding temperature, and the probable error of that mean; uncertainties of timing, frequently contribute nearly a tenth of a second to the probable error. The seventh column contains v, the average speed, reduced to centimeters per second.

Temperature is plotted against speed in figure 1. Observations made at station A are plotted as dots, and those made at station Y as crosses. Two observations of low weight (Nos. L2A and L27A) are plotted as open circles; the recorded temperatures are undoubtedly high, because the timing of ants was begun too soon after introducing the cardboard shade

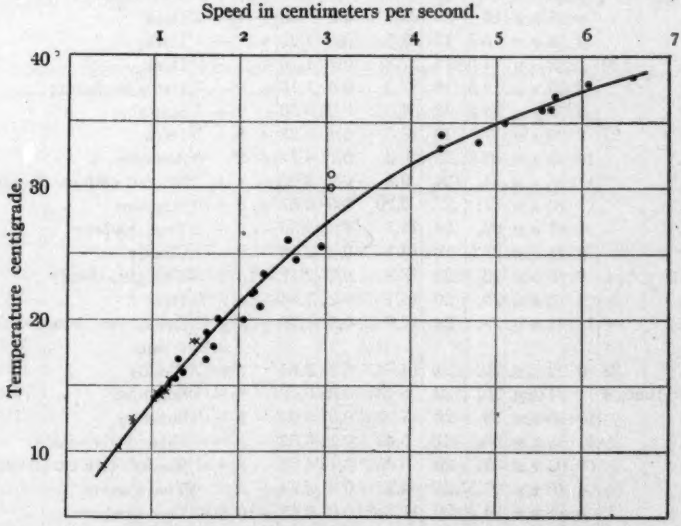

FIG. 1. The temperature-speed curve for *Liometopum apiculatum*.

and the artificial lowering of temperature is responded to much more quickly by the ants than by the mercury thermometer. (Observations made in January 1920 at stations F and G confirm the supposition that the two measures in question were erroneous and should be rejected in a mathematical discussion of the curve.)

The quantities in the eighth and ninth columns of table I are, respectively, the percentage difference in speed for inbound and outbound ants, and the percentage difference in speed for large and small workers. These observations of systematic differences were designed to give evidence of the effect on speed of fatigue and mass. A positive value of $(i - o)/0.01v$

indicates that during that observation ants travelled faster toward the nest than away. A positive value of $(M - m)/0.01v$ indicates a greater average speed for the larger workers. The percentage differences are grouped in table II in order of decreasing temperature.

TABLE I

OBSERVATIONS OF TEMPERATURE AND SPEED

Obsn.	Date (1919)	Time (P. S. T.)	Temp. (Cent.)	No. Ants.	Seconds and P. E.	Speed (cm/sec)	$\frac{i-o}{0.01v}$	$\frac{M-m}{0.01v}$	Remarks
L 1A	May,25	11:20 A.M.	34°,	22	6.9 ± 0.1	4.35	— 4	— 1	Sunshine
L 2A		11:35 A.M.	30,	18	9.8 0.3	3.05	— 5	+ 4	Shaded with cardboard
L 3A		11:46 A.M.	35,	21	5.9 0.1	5.08	— 5	— 3	Sunshine
L12A*		5:25 P.M.	21,	20	13.4 0.3	2.23	+13	+ 6	Tree-shadows
L13A		8:35 P.M.	16	18	22.1 0.2	1.36	+ 4	— 2	Dark
L16A		10:38 P.M.	15.5	17	23.5 0.5	1.27	+17	— 1	Dark
L17A*	26	1:28 A.M.	14.5	15	27.3 0.7	1.10	+ 1	— 2	Dark
L20A		4:00 A.M.	14.5	18	27.2 0.6	1.10	+ 6	— 3	Dawn beginning
L22A		0:55 P.M.	38.5	42	4.55 0.05	6.60	— 9	+ 7	Sunshine
L23A*	27	2:25 A.M.	17,	19	23.5 0.4	1.28	+ 9	+ 5	Dark
L25A*		10:55 A.M.	33.5	26	6.3 0.1	4.77	+ 8	0	Sunshine
L27A*		11:25 A.M.	31,	24	9.8 0.2	3.06	— 4	+ 1	Shaded with cardboard
L28A*		11:40 A.M.	37,	22	5.29 0.08	5.67	+ 6	+10	Sunshine
L29A		4:47 P.M.	26,	24	11.7 0.2	2.56	— 1	+ 2	Tree-shadows
L31A		5:24 P.M.	23.5	18	13.1 0.2	2.29	— 8	— 7	Cloudy
L32A		6:15 P.M.	22,	22	13.8 0.3	2.17	—13	+ 8	Twilight, cloudy
L33A		8:15 P.M.	20,	20	16.1 0.2	1.86	— 6	0	Dark
L35A		11:42 P.M.	18,	24	17.6 0.5	1.70	+12	+ 2	Dark, two hours after rain
L36A	28	9:22 A.M.	20,	24	14.7 0.3	2.04	0	— 7	Cloudy
L38A	June, 4	11:24 A.M.	36,	22	5.43 0.08	5.62	— 6	— 1	Sunshine
L39A		11:40 A.M.	38,	20	4.95 0.07	6.06	— 2	+ 8	Sunshine
L40A		11:53 A.M.	36,	21	5.43 0.13	5.52	— 3	— 3	Shaded with glass
L41A		0:02 P.M.	33,	20	6.95 0.13	4.32	— 3	— 1	Shaded with cardboard
L44A		4:40 P.M.	25.5	25	10.2 0.2	2.94	+ 5	0	Tree-shadow
L46A		5:11 P.M.	24.5	20	11.3 0.2	2.65	+10	+12	Tree shadows
L47A		10:55 P.M.	17,	20	18.6 0.4	1.61	+ 8	— 2	Dark
L48A	5	9:20 A.M.	19,	26	18.5 0.4	1.62	— 9	+ 7	Shadow of wall
L50A*		10:05 A.M.	22,	19	13.9 0.4	2.16	—19	— 2	Shadow of wall
L54Y	Dec. 21	1:40 P.M.	14,	20	29.6 0.9	1.01	+12	+14	Tree-shadows
L56Y		2:16 P.M.	14.5	24	25.8 0.4	1.16	— 8	+19	Weak sunshine
L57Y		3:44 P.M.	12.6	20	39.0 1.1	0.77	— 6	+12	Tree-shadows
L58Y		4:36 P.M.	10.3	20	46.9 1.6	0.64	+27	+ 4	Weak sunshine
L59Y		6:15 P.M.	9.0	16	68.1 2.6	0.44	+11	+15	Dark
L60Y	22	11:42 A.M.	14.5	20	27.0 0.8	1.11	—20	+ 6	Tree-shadows
L61Y		11:56 A.M.	14.5	20	28.5 1.0	1.05	—25	+15	Tree-shadows
L62Y		0:47 P.M.	15.5	22	25.4 0.8	1.18	—14	0	Tree shadows
L63Y		3:40 P.M.	18.0	21	25.9 0.8	1.16	— 9	+ 9	Mottled shadow
L64Y	23	2:47 P.M.	18.6	20	20.2 0.5	1.48	— 2	+ 2	Weak sunshine
L65Y		4:08 P.M.	18.5	20	20.8 0.6	1.44	+13	+ 3	Weak sunshine
L66Y	24	7:15 A.M.	12.5	20	38.7 0.9	0.77	— 4	+ 5	Shadows at sunrise

TABLE II

RELATION OF SPEED TO SIZE AND DIRECTION OF TRAVEL

INTERVAL OF TEMPERATURE	NUMBER OF OBSERVATIONS					Mean of $\frac{i - o}{0.01v}$	Mean of $\frac{M - m}{0.01v}$
	Total	Inbound Workers		Major Workers			
		Fast	Slow	Fast	Slow		
Station A							
38.5° to 30.5°	10	2	8	4	5	−2.2	+1.7
30.5 to 22.5	5	2	3	3	1	+0.2	+2.2
22.5 to 14.5	13	8	3	5	7	+3.5	+0.7
Total range	28	12	14	12	13	+0.8	+1.3
Station Y							
18.6 to 9.0	12	4	8	11	0	−2.1	+8.7

TABLE III

OBSERVATIONS OF TEMPERATURE AND AVERAGE FREQUENCY FOR 4570 ANTS

Observation	Date (1919)	TIME (P. S. T.)	TEMP. (CENT.).	INBOUND		OUTBOUND		Ratio of frequencies N_i/N_o
				Number	Frequency N	Number	Frequency N_o	
L 4A	May 25	0:55 P.M.	37°	150	0.95	150	1.14°	0.83
L 8C		3:46 P.M.	34.5	250	1.58	250	1.33	1.19
L11A		5:15 P.M.	21.5	250	2.05	250	1.27	1.61
L14A		8:40 P.M.	16	150	1.32	150	1.04	1.27
L15A		10:30 P.M.	15.5	100	0.88	100	0.65	1.35
L18A	26	1:37 A.M.	14.5	150	0.80	150	1.21	0.66
L19A		3:59 A.M.	14	250	0.94	250	0.82	1.15
L21A		0:56 P.M.	38.5	125	0.44	125	0.42	1.05
L24A	27	2:35 A.M.	17	125	0.87	125	0.48	1.81
L26A		11:10 A.M.	36	100	0.41	100	0.86	0.48
L30A		5:00 P.M.	25.5	200	1.42	200	1.27	1.12
L34A		8:25 P.M.	20	150	1.14	150	1.36	0.84
L37A	28	9:30 A.M.	19.5	110	0.74	110	0.70	1.06
L45A	June 4	4:51 P.M.	25.5	100	0.85	100	0.85	1.00
L51A	5	10:12 A.M.	23	50	0.20	100	0.76	0.26

In order to determine the relation of number of ants running to time of day and to temperature, the series of observations were made at station A which are recorded in table III and summarized for time of day and temperature in tables IV and V. The frequencies N_i and N_o show the average number of inbound and outbound ants passing a given point on the trail per second—a quantity obtained by dividing the number of ants, fifth and seventh columns, into the intervals of time required for their passage. When the ratio N_i/N_o exceeds unity, more ants are going in

TABLE IV
FREQUENCY AND TIME OF DAY

TIME OF DAY	NUMBER OF OBSERVATIONS	MEAN FREQUENCY		
		In	Out	Ratio
0 to 6 A.M.	3	0.87	0.84	1.04
6 to 12 A.M.	3	0.45	0.77	0.58
0 to 6 P.M.	6	1.22	1.05	1.16
6 to 12 P.M.	3	1.11	1.02	1.09
All	15	0.97	0.94	1.03*

TABLE V
FREQUENCY AND TEMPERATURE

TEMPERATURE (Centigrade)	NUMBER OF OBSERVATIONS	MEAN FREQUENCY		
		In	Out	Ratio
14.5° to 22.5°	8	1.09	0.94	1.16
22.5 to 30.5	3	0.82	0.96	0.86
30.5 to 38.5	4	0.84	0.94	0.90
All	15	0.97	0.94	1.03*

* If the terminal territories of different files do not overlap, this ratio in the long run should be unity or less.

than out. In tables IV and V the quantity N_i/N_o is the ratio of average frequencies for the corresponding intervals of time or temperature; it has a different and better meaning than the average ratio of frequencies for the same intervals.

The following summary of conclusions may be based upon the tables and figure; a discussion of the properties of the temperature-velocity curve s reserved for a later communication.

1. From the measurement of the speed of a thousand individuals of the species *Liometopum apiculatum*, an empirical curve is obtained that for any temperature throughout a range of 30° centigrade gives the speed with an average probable error of 5% for one observation. Conversely, from a single observation of the ant-speed, the temperature can be predicted within 1 degree centigrade.

2. The activity is less erratic for higher temperatures. Thus, the percentage probable error, derived for each observation from the deviations of the individual timings, decreases on the average with increasing temperature and speed from about 4% to less than 2%; but for only one observation does the probable error exceed a millimeter a second.

3. As the temperature rises 30° centigrade the speed changes fifteen fold, increasing uniformly from 0.44 to 6.60 centimeters a second. For a large species of *Eciton*, probably *burchelli*, observed by Beebe in British

Guiana, a normal speed of 7.6 centimeters a second is recorded,[7] and on a later occasion as speed of 15.2 centimeters a second.[8]

4. There appears to be little, if any, real difference in speed towards and away from the nest. Large and small workers have practically the same speed for all temperatures during the summer months; after two months of low temperature, the large workers are conspicuously more active than the small workers (ninth column of table I).

5. This species of ant runs as numerously in the night as during daylight hours; maximum activity falls between noon and midnight, according to the scarcely sufficient observations summarized in table IV. Within the limits of 14° and 38° centigrade, temperature appears to have little effect on the number of ants running in the files.

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[3] Wheeler, W. M., *Proc. Amer. Acad. Arts and Sci.*, Boston, **52**, 1917 (457–569).

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[6] Shapley, Harlow, *Bull. Ecol. Soc. Amer.*, Tuscon, Ariz., **3**, 1919, No. 4.

[7] Beebe, William, *Jungle Peace*, New York, 1919, p. 229.

[8] Beebe, William, *Atlantic Monthly*, Boston, October 1919 (454–464), p. 458.

THE INFLUENCE OF IONS ON THE OSMOTIC PRESSURE OF SOLUTIONS

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I. The osmotic pressure of solutions is usually measured indirectly, namely, by the determination of the freezing-point. Direct measurements have thus far been rare for the reason that it is difficult to prepare membranes permeable for water but not for the solute. The measurements thus far obtained on non-electrolytes by Morse,[1] and by the Earl of Berkeley[2] and their collaborators show that the actual values observed are not far from those expected according to van't Hoff's theory.

The difficulties in the preparation of semipermeable membranes are considerably diminished when we substitute colloidal solutions for solutions of crystalloids. By using solutions of proteins, collodion membranes satisfy the demand of semipermeability.

Gelatin is a very convenient protein for the purpose of such experiments. Like proteins in general, it is an amphoteric electrolyte being capable of forming salts with acids as well as with bases. The hydrogen ion concentration at which gelatin is neither in combination with acid nor with base is $10^{-4.7}N$ ($p_H = 4.7$ in Sörensen's logarithmic symbol). This

hydrogen ion concentration is the isoelectric point. When acid is added to a solution of isoelectric gelatin, part of the acid combines with the gelatin to form gelatin-acid salts, and the proportion of isoelectric gelatin transformed into gelatin-acid salt increases with the amount of acid added. There exists always a definite equilibrium between free acid, gelatin-acid salt, and isoelectric gelatin. Since the physical properties of isoelectric gelatin are different from those of the gelatin-acid salt it is necessary to define a gelatin solution not only by the concentration of gelatin present but also by its hydrogen ion concentration. The writer was able to show that gelatin forms monogelatin salts with many dibasic or tribasic acids, e.g., oxalic, tartaric, and phosphoric acids, while it forms digelatin salts with sulfuric acid.[3]

When we add a base, e.g., NaOH or $Ca(OH)_2$ to isoelectric gelatin, metal gelatinates are formed, and the relative amount of non-ionogenic gelatin transformed into metal gelatinate also depends upon the hydrogen ion concentration.[4]

II. We can measure the osmotic pressure of gelatin solutions by putting them into collodion bags (cast in the form of Erlenmeyer flasks) closed with a rubber stopper perforated by a glass tube which serves as a manometer. A 1% solution of gelatin to which a certain amount of acid or alkali had been added was put into the collodion bag which was dipped into a beaker containing water of the same hydrogen ion concentration as that inside the collodion bag. One per cent solutions of gelatin-acid salts had the highest osmotic pressure when the p_H was about 3.4 and solutions of metal gelatinates had their highest osmotic pressure at a p_H of about 8.4. At about 24° the permanent osmotic pressure was reached in less than 20 hours.

The influence of ions on the osmotic pressure of gelatin solutions was as follows:

(1) The osmotic pressure of 1% solutions of metal gelatinates of a p_H of about 8.4 is a little over twice (but less than three times) as great when the metal is monovalent than when it is bivalent.

(2) The osmotic pressure of 1% gelatin-acid solutions of p_H 3.4 is a little over twice (but less than three times) as great when the acid anion is monovalent than in the case of SO_4.

(3) The addition of neutral salts or alkalies to 1% metal gelatinate solutions of p_H 8.4 lowers the osmotic pressure of the solution and the depressing effect increases with the valency of the cation of the electrolyte added.

(4) The addition of neutral salts or acids to 1% gelatin-acid solutions of p_H 3.4 lowers the osmotic pressure of the solution and the depressing effect increases with the valency of the anion.[5]

This influence of electrolytes on the osmotic pressure of gelatin solutions has no connection with the fact that gelatin is a colloid.[6] This

latter fact plays only a technical rôle in our experiments since it makes it easy to find a strictly semipermeable membrane. The writer has been able to show that the influence of ions on the osmotic pressure of gelatin solutions is connected with the existence of an electrical double layer at the boundary of watery phase and collodion membrane, whereby the water diffuses into the solution in the form of electrified particles.

III. When we separate a solution of a non-electrolyte like cane sugar from distilled water by a collodion membrane, water diffuses into the solution with an initial velocity which is (up to gram molecular concentrations) a linear function of the concentration of the solute. When we separate a solution of an electrolyte from distilled water by a collodion membrane, water diffuses also into the solute but with an initial velocity which for lower concentrations of the solution is no longer a linear but a more complicated function of the concentration and the nature of the ions. The facts observed can be expressed in the following rules:

(*a*) When pure water is separated from a solution of an electrolyte by a collodion membrane, water diffuses into the solution as if its particles were positively charged and as if they were attracted by the anions and repelled by the cations of the solution with a force increasing with the valency and another constitutional quantity of the ion

(*b*) When the collodion membrane has previously been dipped for a short time into a solution of a protein and if the protein solution is then removed Rule *a* holds alsó except that in the presence of hydrogen ions and of simple cations with a valency of 3 or above, in sufficient concentration, the particles of water diffuse into the solution as if they were negatively charged and as if they were attracted by the cation and repelled by the anion of the electrolyte with a force increasing with the valency and another property of the ion.

(*c*) The relative influence of the oppositely charged ions of an electrolyte is not the same for different concentrations of an electrolyte. At lower concentrations the influence of that ion prevails which has the opposite sign of charge as the watery phase, while in higher concentrations (of many, but possibly not of all electrolytes), the effect of the other ion prevails. The turning point lies for a number of electrolytes at a molecular concentration of about $M/256$. Finally a concentration is reached where this ionic influence on diffusion ceases and the gas pressure effect prevails.[7]

These three rules apply also to solutions of gelatin salts. In the case of gelatin-acid salts water is negatively charged and is attracted by the gelatin ion (which is positively charged) and repelled by the acid anion. This repulsion is greater when the anion is bivalent (SO_4) than when it is monovalent, e.g., Cl. Hence the initial rate of diffusion of water into the gelatin-acid solution is greater in the case of gelatin chloride or gelatin nitrate, etc., than in the case of gelatin sulfate.

In the case of metal gelatinate the particles of water diffusing through the membrane are positively charged and attracted by the gelatin anion and repelled by the cation with which the gelatin is combined. This repelling effect is greater in the case of bivalent cations (Mg, Ca, Ba, etc.), than in the case of monovalent cations (e.g., Li, Na, K, NH_4). Hence the initial rate of diffusion of water into solutions of metal gelatinates is greater in the case of sodium gelatinate than in the case of calcium gelatinate. When we add an electrolyte to the solution of a gelatin salt the increase in concentration of electrolyte brings about the depressing effect of that ion which has the same sign of charge as the electrified watery phase, as expressed in Rule *c*.

The permanent osmotic pressure of a solution separated from pure water by a semipermeable membrane is that pressure which has to be applied to the solution side in order to cause equal numbers of particles of water to pass simultaneously through the membrane in opposite directions. It follows from this that the relative influence of electrolytes must be the same on the permanent osmotic pressure as on the initial velocity of diffusion through the membrane from pure water to solution, and this was found to be the case with the solutions of gelatin salts.

IV. The terms "attractive and repulsive effect" of the ion on the charged particles of water served the purpose of simplifying the presentation of the facts observed. In reality the "attractive and repulsive effect" of ions as expressed in Rules *a*, *b*, and *c* are the expression of an additive effect of the oppositely charged ions of an electrolyte on the density of charge of the double layer formed at the boundary of collodion membrane and watery phase. This was proved by experiments on electrical endosmose. When the collodion membrane is bounded on both sides by identical solutions, an equal number of particles of water will diffuse in opposite directions through the collodion membrane. When, however, an external potential difference is produced on the two sides of the membrane, an electrical endosmose will be established to one of the two electrodes according to the sign of the electrification of the watery phase in contact with the collodion membrane. By this method I ascertained the influence of electrolytes on the sign of charge on the watery phase which was expressed in Rules *a*, *b*, and *c*.

Cases of abnormal of osmosis, e.g., the fact that liquid diffuses through parchment membranes from oxalic acid into pure water had been known for more than 50 years and it had been suggested by several authors, Girard, Bernstein, Bartell and Hocker, and Freundlich,[8] that potential differences on both sides of the membrane caused these anomalies. To prove this view it was necessary to measure these potentials. Direct measurements of such potentials have been made by Girard, by Bartell and Hocker,[8] by T. Hamburger,[9] and by the present writer (as yet unpublished), but the results have thus far not been satisfactory. One can,

however, test the idea indirectly by experiments on electrical endosmose. This method of verification seemed to be confronted with a serious difficulty. The influence of ions on electrical endosmose has been investigated by Perrin[10] who found that only one of the oppositely charged ions of an electrolyte, namely, the one which has the same sign of charge as the watery phase, has any influence (namely, a retarding one) on the velocity of endosmose, while the ion with the opposite charge has no influence. In my experiments on common osmose both ions of an electrolyte influenced the rate of diffusion of water simultaneously but in an opposite sense (as stated in Rules *a*, *b*, and *c*). I have investigated the influence of ions on the rate of electrical endosmose and found that Perrin's rule does not hold for collodion membranes, but that the Rules *a*, *b*, and *c* express not only the influence of ions on the transport of water through collodion membranes in common osmosis but also in the case of electrical endosmose.

According to the formula of Helmholtz as modified by Perrin, we know that in the case of electrical endosmose the following relation holds:

$$v = \frac{9.\epsilon.E.D}{4\pi.\eta.l}$$

where v is the quantity of liquid carried electro-osmotically, ϵ is the potential difference between the two strata of the double layer, E the external electromotive force, D the dielectric constant of the medium, η the coefficient of internal friction, and l the distance of the external electrodes.

In our experiments all quantities occurring in the formula except v and ϵ were kept approximately constant. If the theory of Helmholtz is correct, we must, therefore, attribute the influence of ions on the rate of transport of water in electrical endosmose to their influence on ϵ, i.e., the density of charge on the watery phase; and since, moreover, my experiments show that the influence of ions is the same for electrical and for free endosmose through collodion membranes we must conclude that the influence of ions on the initial rate of diffusion and on the osmotic pressure of a solution is due to the influence of ions on the density of charge in the double layer at the boundary of watery phase and membrane.

V. This permits us to define more accurately the influence of ions on osmotic pressure. At the boundary of a membrane and liquid an electrical double layer is formed. In the case of collodion membranes the latter usually assumes a negative charge while the watery phase assumes a positive charge. When the collodion membrane is bounded on one side by pure water on the other by a solution of an electrolyte the anions of the latter increase the negative charge on the membrane and the cations diminish it, both effects increasing with the valency and another property of the ions. At lower concentrations of the electrolyte the influence of the anion increases more rapidly with increasing concentration of the electrolyte than the depressing effect of the cation, while at higher concentra-

tions the reverse occurs. The turning point lies for a number of electrolytes at a molecular concentration of about $M/256$. Positively charged particles of water in the pores or interstices of the membrane will be driven to that side of the membrane which is more negatively charged. Since this is usually the solution side, water will be driven from the side of pure water into the solution. When, however,, the membrane has a higher negative charge on the side of pure water, the flow of the positively charged liquid through the membrane will be from the side of the solution to that of pure water (negative osmosis). This occurs in the case of solutions of $(CaOH)_2$ and $Ba(OH)_2$ when the concentration exceeds $M/256$.

When the collodion membrane has been treated with a protein, it is also generally negatively charged except when the solution contains hydrogen ions or simple trivalent or tetravalent cations beyond a certain concentration (which for H is 10^{-4} N); in this case the membrane is positively and the watery phase is negatively charged. If we add in this case an electrolyte to the water the charge on the membrane is increased by the cations and diminished by the anion of the electrolyte, both influences increasing with the valency of the ion, and another property which is still to be defined. In lower concentrations the influence of the cation increases more rapidly than that of the anion, in higher concentrations of the electrolyte the reverse is true. Whenever the positive charge on the solution side of the membrane is greater than on the opposite side, the negatively charged particles of water will diffuse from the side of pure water to the side of solution. When, however, the positive charge on the solution side of the membrane is less than that of the side of pure water, the negatively charged layer of liquid will flow through the membrane from the side of the solution to that of pure water (negative osmosis). This will happen especially in the case of dibasic or tribasic acids when the concentration exceeds a critical limit. (Since the collodion membrane is permeable for electrolytes we are dealing in reality not with pure water separated by the membrane from solution, but with the condition of weak solution separated from a more concentrated solution.)

It is obvious that with increasing concentrations of the solution the opposite effects of the oppositely charged ions of an electrolyte on the density of charge of the double layer tend to become equal and then the gas pressure effect becomes the main or only driving force for the diffusion of water into solution. For lower concentrations of electrolytes the algebraic sum of both forces, the differences of potential on both sides of the membrane and the gas pressure effect, determine the rate of diffusion of water through a membrane into a solution.

The new experiments on which these conclusions are based will appear in the *Journal of General Physiology*.

[1] Morse, H. N., "The osmotic pressure of aqueous solutions". *Carnegie Inst. Washington, Publ.*, **198**, 1914.

[2] Earl of Berkeley, and Hartley, E. G. J., *London, Proc. R. Soc.* (A), **92**, 1916 (477).

[3] Loeb, J., *J. Gen. Physiol.*, **1**, 1918–19 (559).

[4] Loeb, J., *Ibid.*, **1**, 1918–19 (483).

[5] Loeb, J., *Ibid.*, **2**, 1919–20 (87, 273).

[6] The influence of electrolytes on the osmotic pressure of gelatin is not due to differences in the degree of electrolytic dissociation of the gelatin salts, since, e.g. Na gelatinate and Ca gelatinate of the same concentration of gelatin and hydrogen ions have practically the same conductivity.[4]

[7] Loeb, J., *J. Gen. Physiol.*, **2**, 1919–20 (173, 255); these PROCEEDINGS, **5**, 1919 (440).

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[9] Hamburger, T., *Zs. physik. Chem.*, **92**, 1917 (385); (*Ann. Physik, Beiblätter*, **42**, 1918 (77)).

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EVIDENCE ON THE NATURE OF NUCLEAR ACTIVITY

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Binuclearity hypotheses, founded in part on Richard Hertwig's chromidial hypothesis, have influenced interpretations of extra-nuclear bodies in the cytoplasm profoundly. The obvious dual capacity of the metazoan nucleus, exhibited in kinesis and interkinesis, i.e., in generative (propagatory), and somatic phases, has been made to lend itself to analogy with a true binucleate condition and to an assumption that the nucleus contains two kinds of chromatin. Of these one is supposed to be propagatory (idiochromatin), in evidence at the time of cell division, the other trophic (trophochromatin, somatochromatin), formed by the idiochromatin, but resident in the cytoplasm. This phase of the binuclearity idea is quite independent of that to which support is given by the demonstration of dimorphism in chromosomal groups during oogenesis and spermatogenesis.

The somatic phase of the nucleus covers the period during which it may be assumed that the nuclear enzymes have passed from the nucleus to the cytoplasm, and the cytoplasm has become the seat of synthetic activities. The nucleus at this time is in a "resting" condition; it seems comparatively empty, is acidophile, and basophilic granules may be found in the cytoplasm.

What is the nature of these basophilic bodies? Are they of direct or indirect nuclear origin?

The accounts of the extrusion of chromatin from the nucleus are numerous. Under the influence of the binuclearity hypothesis supposed particles

of chromatin in the cytoplasm have been interpreted as chromidia, or as trophochromidia. Schaxel (1911) in a series of able papers, has presented evidence in support of the idea of emitted chromatin; Dantchakoff's (1916) observations support the idea. On the other hand Beckwith (1914), by no means denying, and Gatenby (1919), strongly supporting the idea of emissions from the nucleus, find no evidence of the emission of *formed* materials, in the sense of filtration of chromatin through the nuclear membrane.

There have been no observations which have enabled us to link, in any satisfactory way, our theories concerning the synthetic activities of the nucleus with the observations of Chambers (1917) on reversible changes in the cytoplasm, and with other knowledge of colloidal solutions. The material on which this paper is based makes an attempt at such a linkage possible.

In 1908 (*Biol. Bull.*, **15**, p. 132), in discussing sections of *Arbacia* eggs fertilized by *Moira* sperm, an inter-ordinal cross, I wrote, "In eggs in which the daughter nuclei are in the resting condition succeeding the first division, the cytoplasm contains many deeply staining rods. The nucleus at this time does not take the chromatin stain and appears like an empty vesicular structure."

"In eggs of the same lot and on the same slides, in which the fibers of the second amphiaster have begun to form, the nucleus again takes the stain and shows the chromatic net, while the cytoplasm is seen to be free from the bodies described."

"These structures have puzzled me not a little, but I have finally reached the conclusion that the eggs in which they occur are degenerating. Even though this be true it is difficult to explain the simulation or perhaps occurrence of longitudinal and transverse divisions of these chromosomes lying free in the cytoplasm."

The facts were so unusual that in 1911 I prepared and later studied a more complete set of material. One of my graduate students, Miss Pauline Shorey, also sectioned and studied duplicate stages, but it is not until recently that I have been willing to venture an explanation.

The material comprises ten stages, taken at short intervals through a period extending from 25 minutes after insemination to 100 minutes after insemination, or through the anaphases of the third division. Unfortunately, from the point of a study of cytoplasmic inclusions in general, but especially of mitochondria, but one fixing fluid, acetic sublimate (5% acetic), was used. Nevertheless, as the straight fertilized Arbacia eggs used for comparison were fixed in the same way, the opportunity for a sound and logical comparison exists. The sections on which the greater part of the study was made were stained in Heidenhain's iron-haematoxylin, but for the determination of special conditions Auerbach's acid fuchsin-methyl green, basic fuchsin, basic fuchsin-methylen

blue, saffranin-gentian violet, thionin, and some of the carmine stains were used.

In the sections of the Arbacia-Moira material a cycle of changes may be noted, changes concerned with the appearance and disappearance of basophilic bodies in the cytoplasm. The development of these bodies may be followed as they appear in a cloud outside of the nucleus, then in lines radiating from the nucleus, then scattered irregularly in the cytoplasm, the nucleus during this time being in the resting condition.

The basophilic bodies appear first in the form of fine granules in the immediate vicinity of the nucleus, then as short rods, which may be seen either singly or in short chains throughout the cytoplasm. These rods are of about the thickness and length of the chromosomes of Arbacia.

A study of the nucleus in the sections of the eggs of this period reveals the fact that the nucleoli have become much paler. In the earlier stages they stain uniformly; in the later the surface (rim in the sections), only stains, while the center remains clear. Deeply stained granules may be seen aggregated at the inner surface of the nuclear membrane as though adsorption were taking place. In some sections a massing of granules may be seen at one side of the nucleus, this mass probably being the expanded chromatin of the male nucleus. There is a distinct impression of diffusion from the nucleus, but there is no evidence of the emission of chromatin as such. At the end of this stage the nucleus is distinctly acidophile.

Then a reversal of the process begins. The sections give the impression of a movement of some of the contents of the cell toward the nucleus; again there is the appearance of a diffuse cloud in the region of the nucleus; the number of rods becomes distinctly smaller, until in the late prophases the cytoplasm has cleared almost completely. The nucleus becomes increasingly basophilic as it passes into the prophases of division. The nucleus increases in size. In the metaphase and in the anaphases of division the cytoplasm is free from rods. During this stage there is no visible evidence of the passage of materials through the nuclear membrane and into the nucleus, but the nuclear content has distinctly increased.

If we correlate these facts with the conditions shown by Chambers to exist in the egg at this time, we find that the diffuse cloud appears while the nucleus is lying in the liquid hyaloplasm sphere. This is at the beginning of the "pause" Wilson (1895), succeeding the fusion of the nuclei in fertilization. The sudden appearance of granules and then of rods indicates a rapid diffusion of enzyme throughout the cell. The rods are very numerous at this time and lie without definite orientation in the cytoplasm. M. R. Lewis determined for Chambers that diffusion of cresyl-blue takes place quickly—in a few seconds—after injection within a cell. The formation of the granules and rods may be interpreted as the result of a reaction of the cytoplasm to the nuclear enzymes.

Following this stage the hyaloplasm sphere divides, as shown by Chambers, and each portion moves and lies like a cap on the nucleus at opposite poles, and the formation of the asters, accompanied by the transition to the gel phase in the cytoplasm, begins. In my material there is evidence of a centripetal flowing, as indicated in the orientation of the rods, as this reversal occurs.

What are these rods? They are not mitochondria. The acetic acid in the fixing fluid has dissolved these. They are not chromosomes. The mass and number of the rods greatly exceed that of the chromosomes. I do not believe them to be chromidia, fragments of chromatin lying freely in the cytoplasm.

I believe that they are a coarse precipitate, possibly a synthetic product, formed by the action of extruded nuclear enzymes into the cytoplasm, and that the processes that I have described are similar to those that occur normally, but which cannot be seen under ordinary conditions, for the reason that the particles ordinarily formed are below the limits of microscopic vision. Nor does there seem to be a probability that they could be distinguished from other moving colloidal particles if the living egg were to be studied with the ultra microscope. It should be emphasized that the granules and rods which are visible in this experimental material and which I have interpreted as a coarse precipitate, are not formed in the straight fertilized eggs.

Turning now to the explanation of the formation of these particles. If we were dealing with solutions of electrolytes the explanation would lie probably in the application of von Weimarn's law on the relation between concentration of the reaction mixtures to the size of the particles of the precipitate. If we compare von Weimarn's microphotographs of the precipitates of barium sulphate formed by pouring together solutions of barium cyanid and magnesium sulphate, with the sections here under consideration, a striking similarity to those obtained at concentrations of 1/500 normal to 1/50 normal will be evident. (Reproductions of some of these microphotographs may be found in Ostwald and Fischer, *Theoretical and Applied Colloid Chemistry*, pp. 27–32.)

But we are dealing with an emulsion colloid, possibly a mixture of suspensoids with emulsoids. The phenomenon that I have described, the formation of granules and rods, is clearly due to a coalescence of particles into coarsely dispersed aggregates.

It seems probable that the suspensoid particles surrounded, "protected," by emulsoid particles, coalesce, as a result of dehydration due to an enzyme emitted from the nucleus. Inasmuch as the change is reversible and these particles pass again into solution the electrical nature of the process is also probable. This naturally would be the case if we were dealing with a suspensoid fraction.

Passing from the changes shown prior to and during the first division,

it should be noted that in this material there is no further formation of rods. The coarsely granular precipitate is well marked in the second and third divisions, but no rods are formed.

It is evident that my conclusion from a study of the material described is that the basophilic bodies found are not in the nature of chromidia, but are the result of indirect nuclear activity. As to the applicability of these results to cases in which basophilic inclusions occur normally, it is impossible to say more than that such cases should be considered in the light of the evidence here given. The explanation offered for the formation of the basophilic extra nuclear bodies described is intended to be suggestive rather than conclusive. It brings together facts which have not hitherto been associated.

A more detailed paper with illustratons is forthcoming.

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A STUDY OF THE PERSISTENCE OF VISION

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Introduction.—It was observed by Allen,[1] while investigating the effect of the color of the light on the persistence of vision, that there seemed to be portions of the retina where the persistence of the retinal impression was less than on the fovea. That is, when no flickering of the color under observation was perceptible in the center of the retina, a slight movement of the eye in any direction which allowed the light to fall upon the peripheral portions of the retina was sufficient to destroy the apparent continuity of the light. Allen attempted to measure the persistence for regions on the temperal side of the retina at 10 and 20 degrees from the axis of the eye but found that the results were "too uncertain to be of any use." The writer has measured the persistence of vision for several colors within the cone whose semi-vertical angle is nearly 40 degrees. More than one hundred points on the retina within this area were observed for each color used. From these data, it is possible to construct a map of the retina showing the persistence of vision for each portion.

Results of this sort should be of interest, not only to the illumination engineer, but to the physiologist and the psychologist as well. If the number of observers were large to insure that the results represent the average eye, it would be possible to construct a map of the retina with "contour lines" to show equal values of the persistence of vision. This was done by the author using the values obtained for his own eyes. The general shape of the lines was found to coincide more or less with the shape of the color fields given by Abney.[2] The extent of the color fields is, of course, dependent upon the intensity of the light. It was not possible to show that the area of the retina covered in this investigation was greater than the color field for the blue for the intensity used. As the color field for the blue is larger in area than for any other color, it seems natural to suppose that the persistence of vision should depend only upon the intensity of the light on portions of the retina outside this area and should be independent of the wave-length.

Description of apparatus.—The persistence of vision was measured by observing the minimum speed at which a sectored disk could be driven without destroying the apparent continuity of the light. The source of light was a concentrated filament incandescent lamp operated at constant voltage. A lens system was used to bring the rays to focus on the sectored disk. When the position of the disk is such that the rays do not strike it, they diverge until they strike a ground-glass screen about 5 centimeters square. An iris diaphragm placed just in front of it makes the size of the illuminated area on the ground-glass adjustable without altering the brightness. The sectored disk, the necessary electric motor to drive it, the incandescent lamp and the lens system are all placed in a light tight box. The eye was then placed 1 meter in front of the ground-glass and a chin rest was provided to insure steady conditions of the retina while making the observations. Needless to say, the investigation was carried on in total darkness. A small electric lamp operated on the storage battery current and carefully shielded was used to read the instruments when necessary. The time for the recovery of the retina after this stimulus was less than the time required to place the apparatus in adjustment for the next reading.

The speed of the disk was measured by means of a small magneto and a voltmeter calibrated to read the speed directly in revolutions per minute. The persistence of vision was first determined for the fovea by causing the disk to rotate at sufficient speed so that no flicker was apparent and then slowly to lose velocity until the first flicker was observed. On the average, it was found possible to determine the critical speed so that subsequent readings would not differ by more than 2 per cent. Observations were also made with the speed of the disk increasing and the average was taken as the persistence measure. A set of filters made by the Wratten

and Wainwright Company was used one at a time when it was desired to use light of a particular color. These filters were found to be very nearly monochromatic. The use of spectrum colors would be more accurate but the intensity of the light cannot be adjusted within as wide limits.

To determine the persistence of vision for off center portions of the retina, a small radiolight sight was used. This was mounted on a slider attached to a long rod and so constructed as to revolve about the center of the diaphragm. In this way it was possible to place the sight in any desired position with respect to the center of the diaphragm. Shallow grooves were placed at intervals along the rod so that it was possible to read the position of the slider in the dark. In the experimental work, readings were taken about every 3 degrees from the center and along directions which made angles with the horizontal of 45, 90, 135, 180, 225, 270 and 315 degrees. The manipulation was the same as before except that the attention was directed toward the radiolight sight and the persistence of vision measured with the light from the ground-glass screen falling on some other portion of the retina. It was, of course, necessary to cover one eye during all of the experimental work.

Experimental results.—Before results could be obtained which were consistent with themselves, it was found necessary to take several precautions. For example, time was given for the eye to become accustomed to the darkness. Results were obtained which showed that 5 minutes in total darkness was sufficient. It was also found that any motion of the body, however slight, would cause the interest to flag. For this reason, the motor controls had to be adjusted so that the motor would change its speed slowly as it was impossible to operate a rheostat by hand. One hand was held on a key which was pressed at the instant that the flicker was seen to appear or disappear and the critical speed noted.

The size of the diaphragm which seemed to give the best results was a circle of diameter 5.84 mm. The persistence of vision is dependent upon the size of the retinal area stimulated and also the scintillation of the light from a small aperture caused more or less uncertainty.[3] The above aperture was chosen as being the smallest that it was practicable to use. With the diaphragm placed at a distance of 1 meter from the eye, the angle subtended by the diaphragm at the eye is 3.36°.

As has already been said, the persistence of vision was determined for several colors and in each case the persistence was measured for about one hundred points on the retina lying inside a circle which is the base of a cone whose semivertical angle is 38.7°. No attempt will be made to give the results in full. They represent the persistence of an impression on the retina of the eye of the author. The eye is known to be normal for color perception but has a moderate amount of astigmatism which should not affect the persistence of vision. A few results will be given to show the nature of the inferences which may be drawn from the investigation.

For red light (6776 A°) the persistence of vision in the fovea was 0.0209 second. The persistence for points lying at equal distances from the fovea was found to be very nearly the same. That is, if lines are drawn showing equal values of the persistence of vision, they appear to approximate circles with the fovea at the center. The deviation from the circle is enough to make them resemble the limits of the color fields for the retina. The circles are in every case flattened so that the major axis of the resulting ellipse is horizontal. The persistence is less for the fovea than for any other part of the retina, and there is a steady increase in the persistence nearly proportional to the distance from the fovea. The maximum value observed occurs on the nasal side of the retina at about 38° from the fovea. The persistence is slightly greater on the nasal side than on the temporal. The maximum value is 0.109 second.

For the yellow-green (5310 A°) very similar results were obtained. The persistence of vision for the fovea is 0.0179 second and is less than any other portion of the retina. The lines of equal values of the persistence are ellipses with the major axes horizontal. The persistence is still slightly greater on the nasal side. The maximum value is observed to occur for the same region as for the red light but the maximum in this case is 0.0339 second showing that the persistence is more nearly constant over the whole retina.

For the blue-violet (4631 A°) the persistence of the fovea is 0.0346 second. There is little change in the persistence for different portions of the retina. The region which gave a maximum value for the red and the yellow-green, now gives a value of 0.0339 second or slightly less than the fovea. The maximum occurs about 7° from the fovea on the nasal side and is 0.0401 second. The minimum of 0.0305 second occurs on the temporal side at an angle of 35° from the fovea. The change between the maximum and minimum amounts only to the difference between $^1/_{25}$ second and $^1/_{35}$ second. For the blue-violet light used, the persistence is very nearly constant over the whole retina.

It will be noticed that these values for the persistence are smaller than those which are sometimes quoted. The values given here represent the time required for the impression on the retina to fade sufficiently to be noticed when compared to a fresh stimulus. They do not represent the time for the total extinction of the retinal image.

The above results were obtained at the laboratories of the Department of Physics at the University of California.

[1] *Physic. Rev.*, **28**, 1909 (48).

[2] Sir William Abney, *Researches in Color Vision*, p. 190, et. seq.

[3] See Abney, *loc. cit.*, p. 181.

A CONTRIBUTION OF GENETICS TO THE PRACTICAL BREEDING OF DAIRY CATTLE

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When the writer began his work at the Maine Agricultural Experiment Station some 13 years ago the only genetic concept which played any part in or influenced the actual operations of the animal breeder was that of phaenotypic selection. The slogan and the practice, so far as there was any intelligent practice, was to "breed from the best to get the best." This was perhaps theoretically a sound enough doctrine, but there was lacking one essential to its practical operation with the anticipated degree of success. This lack, as the rapidly advancing knowledge in the field of genetics soon clearly showed, was of any precise criterion as to how the best *genotypes* were to be selected, when the breeder operated on the basis only of a knowledge of *somatic* performance or characters alone. As a matter of fact it was easy to show, as the writer did show, in the case of egg production in fowls,[1] and milk production in cattle,[2] that the phaenotypic performance was an exceedingly unreliable guide to real breeding worth.

The real value of the measure of an animal as a breeder—as a factor in the improvement of a herd or flock—is the kind of progeny which it produces. Confining the discussion to performance in respect of milk or butter-fat production, a cow may have herself a perfectly enormous record of performance but she is worthless as a *breeder* if her progeny are uniformly poor producers, which can in fact be easily shown to be often the case. Again a bull may come from ancestry of the highest performing ability in respect of milk and fat production, and consequently he may as a calf command a price in five figures, but such ancestry has repeatedly been shown to be no guarantee in fact that his progeny will be great performers.

The paramount importance of having a bull of such genotypic constitution as to insure relatively high dairy production in his offspring, if one is to attain success in the breeding of dairy cattle, is obvious. Nearly 10 years ago the writer began the intensive study of the practical problem of scientifically measuring the breeding worth or value of bulls of the dairy breeds, to the end that the farmer and breeder might operate more intelligently and successfully. It appeared evident that the most important thing which a breeder of dairy cattle desires to know is whether the animals which he breeds are transmitting productive qualities to their offspring. This information is particularly desired for the herd bull since his offspring are much more numerous than those of any single cow and he constitutes one-half the heritage given to them. It would appear beyond question that

if a bull's daughters are on the average poorer milkers, or poorer in the quality of their milk, than the dams from which they came, then the bull is exercising a harmful effect upon the breed. On the other hand, if a bull's daughters are on the average measurably better than the dams from which they came, in productive qualities, then that bull is exercising a beneficial effect on the breed. Stated specifically, if D_d represents the milk production of a daughter of a given bull, and D_m the milk production of the dam of this daughter, then the measure of this bull's transmitting qualities for milk production in respect of this pair would be:

$$\textit{Sire's transmitting power} = D_d - D_m.$$

If the daughter's production is more than the dam's the sign will be plus. Insofar as one daughter-dam pair is indicative the bull is increasing the production of his daughters and consequently improving the breed. If the daughter's production is less than her dam's the sign will be minus. The bull is consequently insofar detrimental to the breed. By the summation of these plus and minus differences with regard to sign, the amount and sign of this summed quantity gives the measure of what he did for all his daughters, and dividing by the number of daughter-dam pairs gives a measure of the mean or average effect of this sire on the breed.

Being convinced that this reasoning was sound, and that if carried out thoroughly the consequences would be bound in the long run to be of incalculable, but great, practical economic value to mankind, the writer inaugurated in his laboratory at the Maine Experiment Station the project of determining the transmitting power as above defined, of every bull in the Jersey breed recorded up to 1916 as having two or more daughters whose milk-producing qualities were known, as well as the producing abilities of the dam. This project involved a very large amount of labor and was interrupted by the war, but has now been finished and will shortly be published.[3] It is the purpose of the present paper to give a brief account of the scope of the work and some of its chief results.

A prerequisite to the comparison of daughter and dam production was an accurate method of correction for change of milk and fat production with age. Considerable study has been given this problem. I showed in 1914[4] that the equation relating milk flow to age in dairy cattle was of the form

$$y = a + bx + cx^2 + d \log x.$$

Since that time these curves have been worked out in my laboratory for all the principle dairy breeds, chiefly by Miner, Gowen, Patterson, and myself. The curve for annual milk production in the Jersey breed is

$$y = 4586.50 + 307.55x - 12.65x^2 + 2216.62 \log x$$

where y denotes milk production in one year in pounds, and x denotes age in years. How closely this equation agrees with the observed facts is shown in figure 1.

It is obviously impossible to present here anything like the total results of this extensive piece of work. As a sample of the sort of results obtained table 1 is shown. This gives the net average change in butter fat production of daughters as compared with dams. The first column of table 1 gives the place occupied by the bull in the series of bulls. The second column gives the name of the bull. The third column states how many

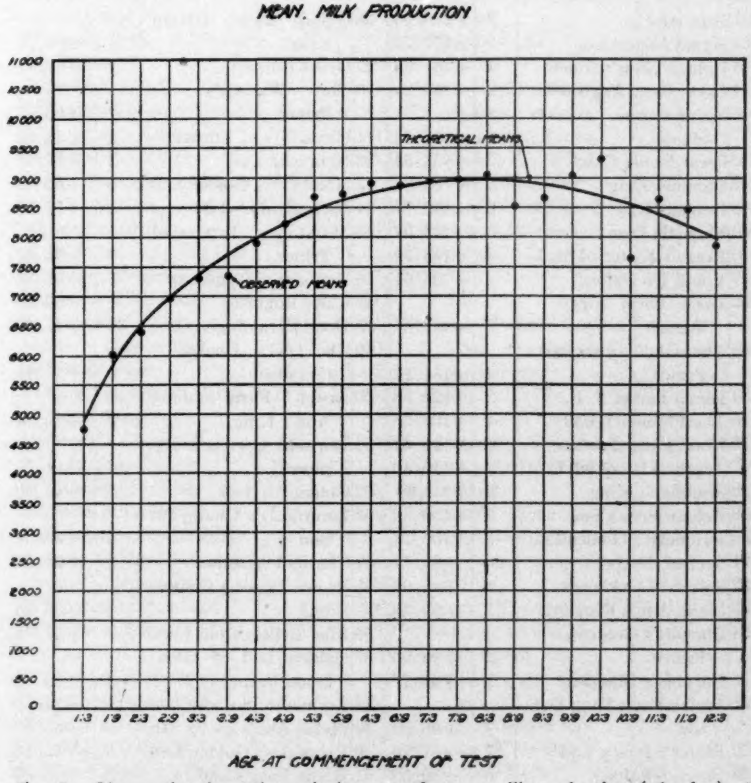

FIG. 1. Observational and theoretical curves of mean milk production (365 day) at various ages in Jersey cows.

pairs of daughter-dam tests the bull under discussion had. The fourth column gives the number of pounds of butter-fat that the bull's daughters are in excess or defect of their dams. It is by this column that the bulls are arranged, the bull whose daughters produced the most butter-fat over the butter-fat production of their dams coming at the top of the list. The plus sign shows the daughters produced more butter-fat than their dams, the minus sign that they produced less butter-fat.

TABLE I

BULLS IN ORDER OF CHANGE IN NET FAT OF DAUGHTERS

NO.	NAME OF BULLS	NO. OF PAIRS	NET CHANGE LBS.	NO.	NAME OF BULLS	NO. OF PAIRS	NET CHANGE LBS.
1	Sans Aloi	2	+210.56	38	Oxford Lass's Cream King	3	+66.23
2	Signal's Successor	3	+177.72	39	Great Edison	2	+62.78
3	Golden Glow's Chief	4	+161.93	40	The Warden's Sir Prince	2	+60.87
4	Hood Farm Pogis 34th	2	+159.24	41	Nova Boy of Turner	2	+58.82
5	Tormentor's Landseer Signal	4	+153.55	42	Nebraska Lad	2	+57.14
6	Clear Brook Chief	2	+153.31	43	Hood Farm Golden Lad	5	+56.50
7	Channel King	2	+150.30	44	Spermfield Owl 2d	2	+51.85
8	Golden Nero	2	+149.77	45	Marston's Interested Prince	9	+51.82
9	Majestic Fern	3	+143.97	46	Eminent's Tormentor	2	+51.81
10	Roma's Rioter of St. L.	3	+143.79	47	Chief Engineer	6	+50.20
11	You'll Do Oxford	2	+136.61	48	Hood Farm Pogis 9th	39	+47.49
12	Hood Farm Figgis Torono	3	+132.50	49	The Owl's Double Grandson	5	+47.45
13	Themisia's Interested Prince	2	+131.12	50	Atwell Farm Melia Ann's King	2	+45.84
14	Rinda Lad of S. B.	2	+126.38	51	Dorinda Carling's Diploma	6	+41.77
15	The Plymouth Lad	4	+123.55	52	Golden Shylock	5	+41.00
16	Hood Farm Torono	34	+121.51	53	Loretta D's Champion Son	5	+38.66
17	Agatha's Brookhill Fox	2	+115.40	54	Taurus Perfection	2	+38.56
18	Fontaine's King	2	+111.60	55	Brown Bessie's Columbus	3	+38.18
19	Golden Fern's Son	9	+109.32	56	Blue Belle's Gold Fern	3	+37.08
20	Baronetti's Golden Lad	2	+100.14	57	Golden Lad of Glenwood Farm	3	+35.08
21	Spermfield Owl	26	+97.71	58	Tormentor Summit Lad	4	+34.39
22	Sultan of Oaklands	3	+95.06	59	Melia Ann's Hero 4th	3	+33.50
23	Melia Ann's King 9th	2	+93.48	60	Keepsake's Golden Lad	5	+32.18
24	Chenille's Golden Prince	2	+93.06	61	Bessie Bate's Lad	2	+30.80
25	Eurybia's Blue Boy	3	+90.17	62	Nutley Tones Owl	3	+30.55
26	Forfarshire's King Dalton	7	+88.00	63	Flying Fox's Oxford Duke 2d	2	+29.30
27	Rioter's Jersey Lad	7	+87.10	64	Lady Letty 4th's Rioter	2	+28.33
28	Queen's Raleigh	3	+82.68	65	Golden Lad of Summit	6	+27.79
29	Pogis 99th of Hood Farm	14	+82.34	66	Golden Alcazar	3	+27.32
30	Royal Majesty	5	+81.34	67	Foxhall's Jubilee	10	+26.64
31	Jenny's Redfern	2	+79.56	68	Eminent's Pilot	3	+24.18
32	Rearguard	3	+79.24	69	The Owl of Meridale	5	+24.17
33	Oxford's Fairy Boy	2	+77.14	70	G. G. Chief of Ashburn	3	+23.75
34	Gertie's Stoke Pogis	3	+70.59	71	Imp. Rozel's Noble	2	+22.76
35	Mabel's Raleigh	3	+70.51				
36	Pogis 66th of Hood Farm	3	+66.86				
37	My Jubilee	2	+66.75				

TABLE 1—(*Continued*)

NO.	NAME OF BULLS	NO. OF PAIRS	NET CHANGE LBS.	NO.	NAME OF BULLS	NO. OF PAIRS	NET CHANGE LBS.
72	Eminent 4th	5	+21.54	112	Mr. Inez Marigold Pedro	3	—13.49
73	Interested Prince	21	+21.07	113	Tonona Pogis	14	—14.00
74	Pogis 75th of Hood Farm	7	+21.00	114	Fontaine's Caiest	5	—15.01
75	Sayda's Heir 3d	15	+20.87	115	Imp. Noble Fontaine Raleigh	5	—15.04
76	Eminent of Acca Farm	3	+20.39	116	Lady Letty's Victor	15	—15.70
77	St. Oner's Torono	2	+20.32	117	Guenon's Lad 2d	2	—15.80
78	Flying Fox's Victor	2	+18.58	118	Oxford's Brigadier	2	—17.48
79	St. Lambert's Ridgewood King	2	+16.32	119	Cinxia's Gamboge Lad	3	—17.69
80	Loretta's King	18	+15.94	120	Portland Prospect	3	—18.98
81	Romulus of Spring Hill	4	+13.34	121	Gamboge's Knight	5	—20.07
82	Cream Princess' Fritz	2	+12.57	122	Oxford Daisy's Flying Fox	6	—20.38
83	Irene's King Pogis	16	+12.31	123	Hood Farm Torono 21st	4	—20.85
84	Anderson's King	2	+11.11	124	Copper Baron	2	—21.24
85	Storrs Golden Lad	2	+9.80	125	Karnak's Noble	4	—21.25
86	The Warden	12	+8.47	126	Jersey Lad's Caspar	5	—21.69
87	Lord Letta of Merridale	2	+7.89	127	Marigold's Exile King	8	—22.37
88	Eva's Grey Fox	2	+7.67	128	Nero's Garfield Lad	2	—22.72
89	Oonan's Count	8	+7.55	129	Alcarano	4	—23.69
90	Valentines Oonan	10	+5.19	130	Oonan's Major	2	—25.33
91	Marigold St. Helier	4	+4.42	131	Merry Miss' Son	5	—26.17
92	Noble of Oaklands	3	+4.24	132	Meridale Interested Prince	6	—26.55
93	Fairfield's Chancellor	2	+3.51	133	Oonan's Conqueror	2	—27.48
94	Exile of the Highlands	4	+3.23	134	Fontaine's Lodestar	4	—27.56
95	Foxy's Brown Poet	7	+2.54	135	Seven Gates	5	—29.25
96	Nimbus of Brondale	3	+1.59	136	Noble's Jolly Golden Lad	2	—29.78
97	Gedney Farm Oxford Lad	2	+1.57	137	Naiad's Golden Lad	3	—31.57
98	Foxy's Fox of Althea	2	+1.52	138	Romeo of Sheomet	4	—31.75
99	Golden Butter Lad	2	+1.25	139	Willson's Exile	2	—33.00
100	Pedro's Vidi	2	—0.51	140	Lad's Oxfordshire	2	—33.39
101	Mabel's Raleigh	3	—1.46	141	Mabel's Poet	5	—36.33
102	Merry Maiden's Grand son	6	—3.27	142	Eminent's Raleigh	2	—37.30
103	Model's Oxford Lad	5	—6.99	143	Reservation Stoke Pogis	5	—37.98
104	Lookout Torono	17	—7.94	144	Fairy Glen's Raleigh	5	—38.32
105	Hood Farm Golden Lad 4th	3	—8.37	145	Gertrude's Jap	2	—38.47
106	Hazel Fern Golden Kin	8	—8.74	146	Rosaire's Olga Lad	13	—38.93
107	Eurybia's Son	3	—9.94	147	Pogis 95th of Hood Farm	5	—39.71
108	Rosetta's Golden Lad	5	—10.04	148	Prince Ramaposa	2	—39.83
109	Pogis of Goliad	3	—12.48				
110	White Oak Monarch	3	—12.64				
111	Prince Noel	4	—13.04				

TABLE I—(*Continued*)

NO.	NAME OF BULLS	NO. OF PAIRS	NET CHANGE LBS.	NO.	NAME OF BULLS	NO. OF PAIRS	NET CHANGE LBS.
149	Valentine's Count	6	—40.75	188	St. Mawes	2	—78.44
150	Sultana's Oxford Lad	3	—40.77	189	Fontaine's Count	2	—80.51
151	Melia Ann's Berkshire	2	—42.39	190	Pedro of Brick House Farm	3	—82.31
152	Glory's Pedro	2	—42.85	191	Jacoba's Emanon	9	—84.27
153	Oxford King of Fernwood	3	—44.37	192	Fad	2	—85.61
154	Gertie's Son's Jamont	2	—45.00	193	Cowslip's Ashley	3	—87.42
155	Pogis Alden Boy	2	—45.04	194	Benedictine King	2	—88.14
156	King Allie Rioter	2	—45.11	195	H. F. Golden Fern's Lad	7	—90.49
157	Hood Farm Torono 20th	19	—45.59	196	Tormentor's Torono	5	—90.51
158	Olga Hazel Pogis	2	—46.97	197	Girldine's Champion Pogis	3	—90.60
159	Marigold's Champion	3	—47.12	198	Kings Rioter Lawrence	2	—94.96
160	Sensational Fern	3	—47.58	199	Gazelle's Fawn Rioter King	3	—98.85
161	Imp. King of Hambie	2	—48.02	200	Noble Peer	4	—98.85
162	Rosaire's Lad of Glenwood	2	—48.79	201	Violet's Oakland Count	2	—99.71
163	Owl's Model Fox	2	—49.41	202	Minaret Exile	9	—100.59
164	Rochette's Noble	2	—49.84	203	Brown Lassie's Compass	5	—100.61
165	Biltmore's Torment	4	—50.61	204	Fort Hill Farm Torono	2	—102.25
166	Gendey Farm Girl's Oxford	14	—53.27	205	Viola's Golden Jolly	2	—102.51
167	Mistletoe Pogis	2	—53.30	206	Oaklands Sultan King	2	—103.95
168	Altama's Pogis	3	—53.31	207	Gertie's Son's Boy	2	—106.18
169	Raleigh's Fairy Boy	10	—53.67	208	Adelaide's Merry Pogis	3	—108.14
170	Quintin Owl	3	—56.07	209	Mona Rose's Glory	5	—110.94
171	Altama Interst	4	—59.39	210	Financial Raleigh	4	—115.05
172	Golden Lucy's Eminent Lad	7	—60.09	211	Jacoba's Premier	3	—115.10
173	Eminent 10th	9	—60.15	212	Magery Golden's Fox	4	—119.76
174	The Imported Jap	11	—61.91	213	Fern's Air	2	—123.55
175	Gamboge's Oxford Lad	2	—64.25	214	Harry B. Gordon	2	—124.46
176	Roving Gipsy	2	—64.75	215	Combination Golden Lad	3	—138.06
177	Golden Laddie of C.	3	—65.75	216	Hector Marigold	2	—144.12
178	Lord North	2	—65.98	217	Lady Mary's Fox	2	—148.02
179	Inez's Stoke Pogis	4	—66.65	218	Rosaire's Golden Lad	2	—149.95
180	Golden Paul	3	—68.05	219	St. Helier of Sheomet	3	—150.59
181	Oxford John D.	3	—68.26	220	Oonan 23d's Grandson	3	—156.92
182	Ben Hur Pogis	2	—68.79	221	Jubilee of Bois d'Arc	3	—162.92
183	Island Lodestar	6	—70.89	222	Oxford Lad's Owl	2	—164.84
184	Daisy's Prince of St. L.	2	—73.38	223	Hood Farm S. Tormentor	3	—219.72
185	Pogis 94th of Hood Farm	2	—75.91	224	Gertie's Son of Washington	3	—227.01
186	Amy's Chief	4	—76.37				
187	Fontaine's Duke	6	—76.54				

This table gives to the breeder information of a sort he has never had before. It enables him at once to form a real judgment of the worth of various bulls which appear in the pedigree of Jersey cattle. Many of the bulls in the minus side of the table have at different times been regarded as leaders of the breed and they and their progeny have sold for fabulous prices. With a scientific measure in hand of their actual effect upon the breed it will in the future be exceedingly difficult to persuade any intelligent breeder of the transcendent merits of these minus sires. On the theoretical side there are many interesting points which flow from this and similar tables, but space cannot be taken here for their discussion.

After presenting the basic tables for milk production, fat percentage, and net fat, similar to table 1 here, the complete paper goes on to discuss various related problems, such as the relation of inbreeding, amount of Island bred stock, etc., to production. The chief results of the whole study may be stated as follows:

1. There are 224 Jersey Registry of Merit sires which meet the requirements of this performance test for their transmitting qualities in milk production. One hundred and five of these sires or less than one-half raise the milk production of their daughters over that of the dams of these daughters. The largest number of daughter-dam pairs is 39 for the sire Hood Farm Pogis 9th, 55[illegible]52. Of those sires which have a large number of pairs, Hood Farm Torono 60326 with 34 pairs stands first in his transmitting qualities raising the milk production of his daughters on the average 2620.1 pounds.

2. Two hundred and twenty-five sires are included in the table giving the sires which met the requirements of the daughter-dam performance test for transmitting qualities of butter-fat percentage. Out of this number 101 sires raised the butter-fat percentage of their daughters' milk as compared with the butter-fat percentage of the dams of these daughters. The leading sire in this butter-fat percentage performance test was Clear Brook Chief 74685 raising his daughters on the average 1.338% of butter-fat. This sire had two daughter-dam pairs. Hood Farm Pogis 9th leads in number of daughter-dam pairs with 42. This bull raised the butter-fat percentage of his daughter on the average of 0.243% over the butter-fat percentage of the dams of their daughters.

3. The sires mentioned as superior in the milk transmitting ability, Hood Farm Torono and Spermfield Owl, do not check up so well in their ability to transmit high butter-fat percentage. Hood Farm Torono caused his daughter on the average to be 0.225 per cent of butter-fat below what the dams of these daughters produced. Spermfield Owl only raised his daughters on the average 0.027% of butter-fat over what the dams of these daughters produced.

4. There are 224 sires of known transmitting ability for net butter-fat. Of this number only 99 sires raise the butter-fat production of their

daughters over that of their dams. The sires which raised the production of their daughters' butter-fat most were Sans Aloi 81012, Signal's Successor 72758, and Golden Glow's Chief 61460. The sires which made the deepest impress on the breed by raising the butter-fat of the largest number of daughters over that of their dams was Hood Farm Torono with 34 pairs and an average increase for each daughter of 121.51 pounds of butter-fat. The next bull, Spermfield Owl, with 26 pairs raised the butter-fat production 97.71 pounds on the average for each of his daughters. Some of the bulls lowering the production of their daughters markedly were Gertie's Son of Washington 83799, Hood Farm S. Tormentor 96311, and Oxford Lad's Owl 75599.

5. The information summarized above was arranged to reveal the transmitting qualities for milk production, butter-fat percentage and butter-fat of Jersey sires to their sons. There were 159 sires which had sons whose progeny performance was known. Of this number 69 or significantly less than half had sons who raised the butter-fat production of their daughters over that of their dams.

6. The sires of superior merit are defined as those which raise the milk production and butter-fat percentage of their daughters as compared with that of their dams. The inferior sires are defined as those sires who lower the milk production and butter-fat percentage of their daughters as compared with the same variables in their dams. The superior sires so defined were arranged by the amount of butter-fat that they increase the production of their daughters over that of their dams. The inferior sires were classified according to the amount of butter-fat that they decrease the production of their daughters in comparison with that of their dams. These two groups of sires are subjected to four generation pedigree analysis to determine their inbreeding and relationship, the amount of Island and American stock, "males and females" and "on the sire's side of the pedigree and on the dam's side of the pedigree," and the individual animals most frequently repeated into two groups of pedigrees.

7. There are 28 sires in the group of sires superior in their transmitting qualities for milk production and butter-fat percentage. In the group of sires inferior in their transmitting ability for these two characters there are 47 sires, a ratio of 1 to 1.7. Such a difference speaks for itself. It emphasizes with startling clearness the need of exact knowledge of the transmitting qualities of bulls to be bred as sires and of the necessity for exact knowledge of the inheritance of milk production and butter-fat percentage.

8. The inbreeding coefficients show that the sires of superior merit have 7.08 % of the greatest possible inbreeding up to the fifth generation. The inferior sires are inbred 9.65% of the greatest possible amount (continued brother and sister mating). The group of sires poorer in their transmitting

qualities are consequently more inbred than the group of sires with superior transmitting qualities.

9. The analysis of the pedigrees for the amount of relationship that may exist between the sire and dam of the individual bulls in the superior group and in the inferior group shows that there is little or no difference in the amount of this relationship within the two groups.

10. The resolution of the four generation pedigrees into the Island bred Jerseys and by difference into the American bred Jerseys showed the mean number of Island males in the pedigrees of the superior sires' group to be 8.07 and the mean number of females 7.79. The mean number of Island bred males in the inferior sires' group were shown to be 6.94 and the mean number of females 6.55. The group of sires which increased the production of their daughters over that of their dams had, consequently, more Island bred stock in their pedigrees. The females in each group of the pedigrees had a smaller proportion of Island bred individuals than the males had in each of the groups.

11. Study of the pedigrees of these two groups of sires discloses the fact that all the animals which appeared in the pedigrees of the superior sire on the male side of the pedigrees more than four times or on the female side of the pedigree more than three times also had appearances in the pedigrees of the sires inferior in their transmitting qualities. This fact alone makes it clear that the appearance of certain famous animals in pedigree of a given bull is no guarantee of that particular bull's worth.

My former student and successor at the Maine Experiment Station, Dr. John W. Gowen, expects presently to publish similar investigations for the other chief dairy breeds. It is a great pleasure to acknowledge the faithful and painstaking help of my two assistants, Mr. John Rice Miner and Dr. John W. Gowen in bringing this investigation of the Jersey breed to completion. Without their aid it is doubtful if this would ever have been finished, particularly in view of the influences, which have operated on my own scientific work during the past three years.

[1] Pearl, R., and Surface, F. M., Studies on the Physiology of Reproduction in the Domestic Fowl II. Data on the Inheritance of Fecundity obtained from the Records of Egg Production of the Daughters of "200-Egg" Hens. *Maine Agric. Expt. Stat. Ann. Rept.*, 1909, pp. 49–84. Also Pearl, R., and Surface, F. M., Is there a Cumulative Effect of Selection? *Zs. Ind. Abst. Ver.*, **2**, 1909 (257–275).

[2] Pearl, R., Breeding for Production in Dairy Cattle in the Light of Recent Advances in the Study of Inheritance, *Eighth Ann. Rept. Comm. Agr. Maine*, 1910, pp. 118–129.

[3] The complete paper will have the following bibliographic status: Pearl, R., Gowen, J. W., and Miner, J. R. Studies in Milk Secretion, VII. Transmitting Qualities of Jersey Sires for Milk Yield, Butter-fat Percentage and Butter Fat, *Maine Agr. Expt. Stat. Ann. Rept. for* 1919, pp. 89–204.

[4] Pearl, R., On the law Relating Milk Flow to Age in Dairy Cattle, *Proc. Soc. Exp. Biol. Med.*, **12**, 1914 (18–19).